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A variable charting statistic for monitoring the covariance matrix of bivariate processes

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Abstract. This article proposes a control chart to monitor the covariance matrix of bivariate processes using a mixture of attribute and variable charts, namely MIX-VAR chart. The items are first classified as approved or disapproved, using devices such as a go-no-go gauge. If the number of disapproved items is equal or greater than a control limit, the units of the next sample are measured and their sample variances are calculated. The proposed chart performs better than its competitor attribute chart, but not better than its competitor variable chart. However, it is more economically advantageous than the variable inspection.

Keywords: control chart, variable, attribute.

1 Introduction

Control charts were created by Shewhart in the 1920s to monitor quality characteristics. They can be by variables or by attributes.

Variable control charts are used to monitor continuous quality characteristics. When measuring a quality characteristic is expensive and time-consuming, it is interesting to assess the possibility of using a control chart based on attribute inspection to monitor the process mean or variance. On the other hand, variable control charts generally require a much smaller sample size than the corresponding attribute control chart [1].

Most control charts proposed for monitoring the covariance matrix of bivariate processes, use variable inspection. see [2]. The authors of [3], investigated the performance of control charts by attributes for monitoring the covariance matrix, including the MAX D chart. When compared to the generalized variance chart | S |, the MAX D chart is faster at signaling the special cause for any magnitude of disturbance, except when the variables are highly correlated. In [4], the authors proposed a control chart based on the MIX strategy for monitoring the mean vector of bivariate process, the ATTRIVAR chart (attribute and variable).

This article proposes a control chart to monitor the covariance matrix of bivariate processes using a mixture of attribute and variable charts, namely MIX-VAR chart.

2 The Proposed chart

We assume that the random vector (X, Y) follows a bivariate normal distribution with mean vector $\boldsymbol{\mu} = (\mu_X, \mu_Y)^T$ and the in-control covariance matrix:

$$\boldsymbol{\Sigma}_0 = \begin{bmatrix} \sigma_X^2 & \rho\sigma_X\sigma_Y \\ \rho\sigma_X\sigma_Y & \sigma_Y^2 \end{bmatrix} \quad (1)$$

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The assignable cause increases the variance of the quality characteristic X and/or the variance of the quality characteristic Y, without changing the correlation ρ and the mean vector $\boldsymbol{\mu} = (\mu_X, \mu_Y)^T$. When the process is out of control the covariance matrix is:

$$\boldsymbol{\Sigma}_1 = \begin{bmatrix} a_1^2\sigma_X^2 & \rho a_1 a_2 \sigma_X \sigma_Y \\ \rho a_1 a_2 \sigma_X \sigma_Y & a_2^2 \sigma_Y^2 \end{bmatrix} \tag{2}$$

The constants a_1, a_2 denote the shift sizes respectively for the standard deviations σ_X and σ_Y .

At each inspection, a sample of size n must be available. The sample units are submitted to go/no-go gauge tests. The statistic plotted in the Max D chart is D= max (D_x, D_y). being D_x (D_y), the number of disapproved items with respect to the X (Y) quality characteristic. If D is lower than the control limit CL_D, the process is declared to be in-control; otherwise, the X and Y quality characteristics of the next sample are measured, and the X and Y observations are used to obtain the V value, where V is the maximum of two sample variances. The process is declared to be out of control (in control) when the V point is plotted above (below) the control limit.

3 The Performance of the Proposed Chart

In this section the performance of the proposed control chart is presented. To study the performance, we use the Average Run Length, *ARL*. The *ARL* measures the speed with which the control chart signals. In this paper, the value of the in control *ARL*=370.4. Table 1 presents the ARLs for the proposed chart with ρ=0.0; 0.3 and 0.5 (n=5, CL_D=0.5). The percentage %Vmax is the in-control rate with which the *X, Y* quality characteristics of the sample items are measured. It is relevant to annotate that ARL_0 of 370 is not reached with ρ>0.5 and %Vmax=33.3%.

Table 1 The ARLs of the MIX-VAR charts (%Vmax=33.3%)

a_1	a_2	0	0.3	0.5
1	1.25	37.2	39.9	40.3
1	1.5	10.6	9.69	9.53
1	1.75	4.17	4.35	4.26
1	2.0	2.61	2.65	2.71
1.25	1.25	31.4	32.2	31.5
1.25	1.5	10.6	10.8	10.8
1.25	1.75	4.75	4.87	4.70
1.25	2.0	2.83	2.90	2.82
1.5	1.5	7.35	7.62	7.96
1.5	1.75	4.31	4.33	4.47
1.5	2.0	2.71	2.81	2.82
1.75	1.75	3.25	3.32	3.49
1.75	2.0	2.36	2.45	2.55
2.0	2.0	1.96	2.02	2.08

From Table 1, we observe that the ARL usually increases with ρ, except when a_1=1.25. In this case, the chart´s performance is worse when ρ= 0.3.

Table 2 presents the ARLs for the proposed chart varying the %Vmax (n=5, CL_D=0.5).

Table 2 The ARLs of the MIX-VAR charts

a_1	a_2	66%	33%	15%
1	1.25	37.7	37.2	47.8
1	1.5	9.00	9.23	10.4
1	1.75	4.10	4.17	4.65
1	2.0	2.50	2.61	2.91
1.25	1.25	23.5	31.4	45.3
1.25	1.5	8.66	10.6	13.5
1.25	1.75	4.15	4.75	5.54
1.25	2.0	2.52	2.83	3.21
1.5	1.5	5.64	7.35	10.6
1.5	1.75	3.38	4.31	5.62
1.5	2.0	2.34	2.71	3.39
1.75	1.75	2.55	3.25	4.33
1.75	2.0	1.97	2.36	3.01
2.0	2.0	1.66	1.96	2.46

The chart is always faster to signal for higher levels of the in-control rate with which the *X, Y* quality characteristics of the sample items are measured.

Table 3 was built to compare the current proposal with the main competitors (n=5, CL_D=0.5, ρ= 0 and %Vmax=33.3%).

Table 3 The ARLs of the Max D, Vmax and MIX-VAR charts (ρ=0)

a_1	a_2	Max D	Vmax	MIX-VAR
1	1.25	52.8	35.9	37.2
1	1.5	13.7	8.46	10.6
1	1.75	6.03	3.79	4.17
1	2.0	3.57	2.37	2.61
1.25	1.25	28.6	19.1	31.4
1.25	1.5	11.4	7.12	10.6
1.25	1.75	5.57	3.54	4.75
1.25	2.0	3.43	2.29	2.83
1.5	1.5	7.24	4.54	7.35
1.5	1.75	4.45	2.86	4.31
1.5	2.0	3.03	2.04	2.71
1.75	1.75	3.31	2.19	3.25
1.75	2.0	2.51	1.74	2.36
2.0	2.0	2.08	1.5	1.96

The proposed chart performs better than the MAXD chart, but not better than the Vmax chart. However, it is more economically advantageous than the variable inspection.

Let us assume that the costs c_1 and c_2 with ($c_1 < c_2$) are respectively the cost to evaluate a unit by attribute and variable inspection. Thus the average sampling cost (ASC) for the control charts of Table 3 are respectively

$$ASC_{Max\ D} = nc_1;$$
$$ASC_{Vmax} = nc_2;$$
$$ASC_{MIX-VAR} = nc_1(1 - p_2) + nc_2 p_2; \quad (3)$$

being p_2=%Vmax. Expressing $c_2 = kc_1$, where $k > 1$ is a constant and substituting in (3), they result

$$ASC_{Max\ D} = nc_1;$$
$$ASC_{Vmax} = nc_1 k;$$
$$ASC_{MIX-VAR} = nc_1(1 - p_2 + kp_2); \qquad (4)$$

Clearly from (4), the average sampling cost of Max D chart is the lowest one among the control charts in Table 3 and $ASC_{MIX-VAR} < ASC_{Vmax}$ once (1- p_2+k p_2) < k; that is, the MIX-VAR chart will be always averagely cheaper than the Vmax chart, even if p_2 is high. To illustrate, if n=5, c_1=1, k=2.0 and p_2=0.5, the $ASC_{Max\ D}$=5.0; the ASC_{Vmax}=10.0 and the $ASC_{MIX-VAR}$=7.5.

Table 4 shows the percentage increases in the ARL (%ARL) and the percentage decreases (in three cases, increases) in the ACS (%ACS) comparing the MIX-VAR with the Vmax charts considering Table 3 and expression (4).

Table 4 The %ARL and the %ASC for the VMAX and MIX-VAR charts

a_1	a_2	%ARL	%ASC
1	1.25	3.49	-45.1
1	1.5	20.2	-20.0
1	1.75	9.11	-36.7
1	2.0	9.20	-36.6
1.25	1.25	39.2	8.62
1.25	1.5	32.7	-1.20
1.25	1.75	25.5	-12.1
1.25	2.0	19.1	-21.7
1.5	1.5	38.2	7.11
1.5	1.75	33.6	0.21
1.5	2.0	24.7	-13.2
1.75	1.75	32.6	-1.33
1.75	2.0	26.3	-10.9
2.0	2.0	23.5	-15.1

Table 4 helps the users to choose the chart according to their priority: performance or cost. Example: if a_1=1 and a_1=1.5, the Vmax chart is approximately 20% faster than the MIX-VAR chart; however, the MIX-VAR chart is approximately 20% cheaper.

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References

Aparisi F. Ho. L.L. (2016) ATTRIVAR: Optimized control charts to monitor process mean with lower operational cost. International Journal of Production Economics 182: 474-483

Machado MAG. Costa. AFB. (2008) The double sampling and the EWMA charts based on the sample variances. International Journal of Production Economics 114:134-148

Machado MAG. Ho LL. Costa AFB. (2018) Attribute control charts for monitoring the covariance matrix of bivariate processes. Qualilty Reliability Engineering International 34: 257–264

Montgomery. D. C. (2009) Introduction to statistical quality control. John Wiley & Sons.