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Markovian study of biometric and social states of the career flow of military personnel: a contribution to the Military Social Protection System of Brazilian Armed Forces

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Abstract Due to the particularities of the military profession, for good governance and internal control practices to be adopted, it is necessary that the military career be analyzed according to its specificities. In the present work, a Markovian study is carried out in relation to the career flow of personnel from the Brazilian Armed Forces. It is intended to verify whether it is possible, from the biometric and social states to which a military man may be subject, to achieve a state of stability. A simplified modeling with six possible states was arbitrated in order to guarantee the conditions for applying the Chapman-Kolmogorov equation. A 2018 Brazilian Navy database was used to estimate the assumption probabilities for each state. This article provides a brief review of the Markov Chain concepts and the results indicate that this technique is applicable to the sample in question, being possible to calculate the steady-state probabilities.

Keywords: Military Social Protection System; Markov Chain; Brazilian Armed Forces; Steady state probability; Career flow.

1 Introduction

The military career of the Armed Forces (AF) has its own characteristics and is distinguished from other state careers. Even compared to the federal public service, military personnel are subject to distinct rules and are exposed to very specific risks and working conditions.

In the civilian career, pension funds are responsible for mitigating the risks to which employees are subject, in addition to guaranteeing earnings when the individual reaches old age or stops working (SUKONO et al., 2018).

Unlike other professional areas, the Brazilian military, following National Constitution promulgated in 1988, is not covered by any of the social security schemes. Also, according to Constitutional Amendment 18/1998, the system in which the military is linked is defined as a “Constitutional Regime” with its own characteristics. This regime called the Military Social Protection System (Sistema de Proteção Social dos Militares -SPSM – in portuguese) is a fundamental compensatory social contract for the existence of AF (LEAL et al., 2019).

Ferreira (2017) differentiates SPSM of Brazil’s Armed Forces, that is, Brazilian Navy (BN), Brazilian Army (BA) and Brazilian Air Force (BAF), from social security regimes. For that author, the SPSM aims

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to ensure dignity for the military and dependents, due to the peculiarities of the military profession. Santos (2018) adds that the military SPSM consists of an integrated set of permanent and interactive legal instruments, in their aspects of remuneration, social assistance and health, which aim to guarantee the protection and dignity of the FFAA military and their dependents, given the peculiarities of the military career.

At the same time, as in other professional areas, the Armed Forces are urged by government control agencies to account for their spending and to ensure the proper use of public resources. In order for a correct governance practice to be possible, through such specificities of the military of the armed forces, it is necessary that this portion of the population is carefully analyzed.

The issues surrounding AF are not simple and go beyond the social and economic spheres. The concept of a system is related to a set of elements or components that interact to achieve a goal (PEREIRA et al, 2015). In this case, this system should be concerned with ensuring relative peace of mind so that the military can take the risk inherent in their professions, knowing that their families are protected, including especially a decent retirement and an effective pension system. In this context, an important issue to be addressed is the understanding career flow of the military of the Armed Forces.

From the moment an individual joins Brazil's Armed Forces, the military becomes subject to different biometric and social states. These states are independent and can be represented by a graph. These different states interact according to the individual's probability of passing through each state, being able to change to any other state or remain in the same state in the next instant.

Industrial Engineering and Operations Management, operational research is a multidisciplinary science that makes use of mathematical tools to solve real problems, assisting the decision-making agent to make decisions about complex problems (TEIXEIRA, 2020; TEIXEIRA et al. 2019). In this paper, the authors made use of the Markov Chain tool.

Regarding the modeling problem, it is assumed that the Markov holds property is one of the few simple methods of introducing a statistical dependency in a model of a stochastic process in such a way that it allows the strength of the dependency at different lags to decline as the lag increases (BON; ISAH, 2018).

Studies applying Markov Chain theory are common in academic literature. A search carried out in May 2020 with the keyword "Markov Chain" in the Scopus and Web of Science databases found, respectively, 87,371 and 17,617 articles published since 2010. However, when performing a search in the same databases with the keyword "Markov Chain" individually combined with the expressions "military career" or "career flow" or "military career flow" or "biometric states" does not return any results for published articles. This may indicate a gap in the literature when combining Markovian studies with research related to the military's career flow.

The objectives of this paper are to verify if the probabilistic decreases to which an active individual is subjected tend to some convergence state and to calculate, if any, the probability of stable state for the biometric and social states in which an active individual may be. These results may assist the military authorities in planning the career flow, as well as in the budget forecasts for expenses with active and retired personnel. In addition, it is expected that the present study will enable other researchers to replicate the methodology in other areas of the public and private functions, contributing to social security studies.

2 Theoretical Foudation

Markov process is defined as a stochastic process in which only the current value of the random variable is relevant in the future forecast (SPITZER, 1970; D'ARCY, 1989). The previous values, in addition to the most recent, therefore do not affect the subsequent values (PRAVEEN; ANAND, 2019). Stochastic processes, in turn, are mathematical models of random phenomena that evolve according to the prescribed dynamics (SERFOZO, 2009). The assumption of Markov Chains is, therefore, the independence of the present in relation to the past. Thus, knowledge of the state at any moment would be enough to predict the future (GOMES; GOMES, 2019). The fundamental properties of a Discrete-Time Markov Chain (DTMC) are Joint Probability Distribution, Chapman-Kolmogorov Equation, and Steady-state Probabilities (LIU; HASAN; TAHAR, 2011).

A graph is an element of abstraction that represents a non-empty and finite set of elements, called nodes, and their interdependent relationships, called edges (GOMES; RIBEIRO, 2014). A Markovian process can be represented by means of graphs, where the nodes represent the states and the edges indicate the probabilities of transition between these states (RESTUM, 1999). Figure 1 illustrates a two-state Markov process, where p_{ij} represents the probabilities .

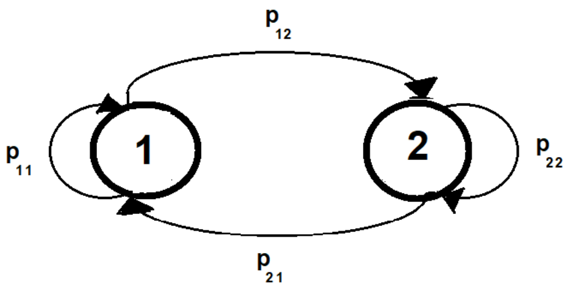

Fig. 1. Graph of a two-state Markov process. Source: Authors

For Seneta (1996), other common way of representing the transition probabilities in n steps in a Markov Chain is through a transition matrix, where the state represented in the side column occurs to the state represented by the upper line. Figure 2 represents an example of transition matrix formation.

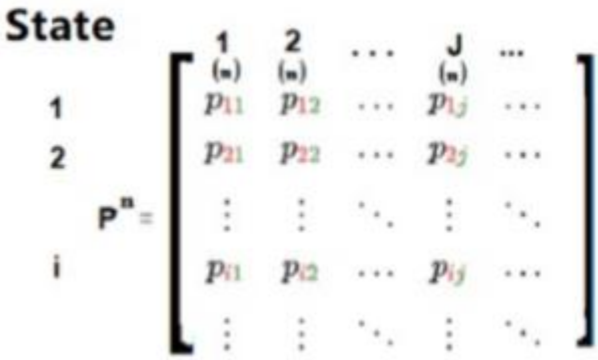

Fig. 2. Example of transition matrix formation. Source: Adapted from Seneta (1996)

Let P be a matrix k × k with elements $\{P_{i,j}: i, j = 1, ..., k\}$. A random process $(X_0, X_1, ...)$ with finite state space $S = \{S_1, S_2, ..., S_k\}$ is said to be a Markov Chain with transition matrix P, if ∀n, all i, j = {1, ..., k} and all $i_0, ..., i_{n+1} = \{1, ..., k\}$, then:

$P (X_{n+1} = S_j \mid X_0 = S_{i0}, X_1 = S_{i1}, ..., X_{n-1} = S_{in-1}, X_n = S_{in}) = P (X_{n+1} = S_j \mid X_n = S_i) = P_{i,j}$ (Equation 1)

Although Markov chains represent one of the simplest mathematical models in terms of random phenomena involving time, their application is very wide, being considered the most important example of a stochastic process (ABENSUR, 2013). According to Hillier and Lieberman (2013), the lasting properties of a Markov process depend on the characteristics of its states and its transition matrix. Thus, the states of the Markov Chain could be classified into: (a) Accessible, from another “y” state, whenever possible for the system enters that state when it starts in the “y” state. If an “x” state is accessible from state “y” and vice versa, states “x” and “y” are said to communicate. Two communicating states are said to be of the same class. When all the states of a given Markov Chain communicate this is said to be irreducible; (b) Transient, when after reaching this state there is a possibility that the process never return to this state again; (c) Recurrent, when after reaching this state there is certainty that the process return to this state again; (d) Absorbent, when after reaching this state there is certainty that the process will never leave this state again;

and (e) Aperiodic, when, with two consecutive numbers “t” and “t + 1”, the process can be in state “x” at instants “t” and “t + 1”.

A Markov chain is said to be ergodic if all its states are ergodic states, that is, simultaneously recurrent and aperiodic, in a set of finite states. That is, a transition matrix in which the states are revisited and the states can remain unchanged in two consecutive moments. According to Chapman-Kolmogorov equations, the transition probability matrix in “n” steps can be obtained by calculating the nth power of the transition matrix in a given step. It means that a process currently in state “i” will be in state “j” after “n” transitions according to probability of theorem:

$$P_{ij}^{(n+m)} = \sum_{k=0}^{\infty} P_{ik}^{(n)} P_{kj}^{(m)} \quad \text{(Equation 2)}$$

When a Markov Chain is ergodic and irreducible, there is a limiting probability, after a sufficiently large number of transitions between states, that the system will be in a certain “x” state, no longer depending on its initial state. Steady state probability is called this condition in which a Markov Chain reaches an unchanged transition matrix. It is noteworthy that the probability of steady state does not mean that a given system no longer transitions between its states, but that the probabilities involved in the process do not change anymore.

3 Methods

The first author to study the military career of the Brazilian armed forces from an approach to biometric and social states was Santos (2018). However, in that work he opted only to present an analytical view of which states the individual could be framed in, without going into details about changes in state and their probabilities.

In the present work, a new model is proposed based on the analysis of existing functional situations. It was decided to perform a new arrangement of decreases in biometric and social states, disregarding the absorbing states and obtaining a simplified model. In this new graph, the interaction between biometric and social states was privileged, focusing on the dependents of the military and their entry or exit from active service. In this new approach, the possibility of a military man not changing his biometric and social status between an instant “t” and the next “t + 1” is also considered. In this way, 6 biometric and social states were integrated, namely: (1) Active Military without dependents; (2) Marriage; (3) Paternity; (4) Retirement (with possibility of recall); (5) Definitive Retirement / Disability (Temporary or Not); (6) Resignation or Discharge. The different biometric and social states from which a military servant can be classified are illustrated in the Figure 3.

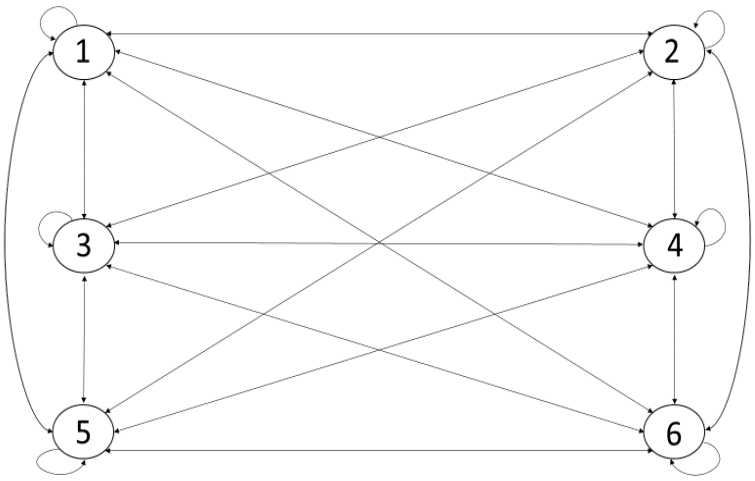

Fig.3. Biometric and social states to which a military servant may be subject. Source: Adapted from Teixeira (2020).

The graph in Figure 3 shows that an active individual is subject to joint probabilities of retiring or invalidating himself. Each vertex of the graph establishes a state in which the military can meet. It is often said that this individual is subject to probabilistic multi-increments. From the Biometric States of Figure 3, we obtain the transition matrix that represents the relationship between the state changes, where pi.j indicates the probability of changing from state "i" to "j". If a given state "j" was not accessible from state i, we would have pi.j = 0 (zero). Figure 4 illustrates transition matrix with six biometric states.

	1	2	3	4	5	6
1	p1.1	p1.2	p1.3	p1.4	p1.5	p1.6
2	p2.1	p2.2	p2.3	p2.4	p2.5	p2.6
3	p3.1	p3.2	p3.3	p3.4	p3.5	p3.6
4	p4.1	p4.2	p4.3	p4.4	p4.5	p4.6
5	p5.1	p5.2	p5.3	p5.4	p5.5	p5.6
6	p6.1	p6.2	p6.3	p6.4	p6.5	p6.6

Fig. 4 Transition Matrix with six Biometric and social states to which a military servant may be subject. Source: Authors

Probabilities of changing from different states were obtained from the 2018 Brazilian Navy Statistical Yearbook (Anuário Estatístico da Marinha do Brasil - ANEMAR 2018 – in portuguese). The Navy Statistical Yearbook is an annual publication, which provides an overview of the Brazilian Navy, with statistics information on operating and administrative activities for the year in question. It also presents historical series, which allow the comparability of information and monitoring of evolution, portraying various periods of Naval History. With the identification of behaviors and patterns of data presented in this document, the Naval High Administration has a tool to support decision-making processes. Based on public data from ANEMAR 2018 it was possible to estimate the transition probabilities between biometric states. Values were estimated by the ratio of BN military numbers in each state in 2018. Figure 5 represents the Matrix of estimated probabilities transition between each Biometric and social state.

	1	2	3	4	5	6
1	1	0,1364	1,3745	0,532	1,0768	0,0048
2	0,7338	1	1,0088	0,39	0,7902	0,0035
3	0,7275	0,9913	1	0,387	0,7834	0,0035
4	1,8811	2,5632	2,5857	1	2,0255	0,0091
5	0,9286	1,2654	1,2766	0,494	1	0,0045
6	207,0128	282,08	284,5577	110,1	222,91	1

Fig. 5. Matrix of estimated probabilities transition between six Biometric and social states to which a military servant may be subject. Source: Authors

4 Discussion and Results

Making a decision is making a choice among several alternatives. Efficiency in decision making consists in choosing the alternative that, as much as possible, offers the best results. The structuring of the model is fundamental in a decision support process, which has a mixed character between science and art (CARDOSO et al, 2009). The model proposed by the graph of the relationship between biometric and social states is able to represent interactions between different situations in a simple, efficient and effective way. It is observed that the matrix of figure 4, which represents the graph of figure 1, is ergodic, as it satisfies the recurrence and periodicity conditions. That is, in this new simplified graph it is possible, by applying the Markov Chain, to obtain the steady state probability.

The values in Figure 5 have been normalized so that they fall within a range between 0 and 1. According to Gomes and Gomes (2019) there are four different ways of normalization in academic literature as presented in Table 1. Let a_{ij} be a term of a set of elements to be normalized.

Table 1 Normalization Process

Procedure	Formula
1	a_{ij} / Max a_{ij}
2	(a_{ij} – Min a_{ij}) / (Max a_{ij} – Min a_{ij})
3	a_{ij} / $\sum a_{ij}$
4	a_{ij} / ($\sum a_{ij}^2$)

The authors decided by Procedure 3. This normalization consists of dividing the number to be normalized by the sum of the subset numbers. This ensures that all values are non-null and are within the range 0 to 1. Figure 6 represents the transition matrix with values already normalized.

	1	2	3	4	5	6
1	0,242477	0,0331	0,333285	0,129	0,2611	0,0012
2	0,186889	0,2547	0,256927	0,099	0,20125	0,0009
3	0,186903	0,2547	0,256911	0,099	0,20126	0,0009
4	0,186903	0,2547	0,25691	0,099	0,20125	0,0009
5	0,186886	0,2547	0,256923	0,099	0,20126	0,0009
6	0,1869	0,2547	0,25691	0,099	0,20125	0,0009

Fig. 6 Matrix normalized of estimated probabilities transition between six Biometric and social states to which a military servant may be subject. Source: Authors

Using the Chapman-Kolmogorov equation and raising the transition matrix of the 6state Markov Chain to the 54th power, it was possible to calculate the steady state probability. Figure 7 illustrates the probability matrix after the 54th application of the Markov Chain.

	1	2	3	4	5	6
1	0.197891869	0.21082041	0.272027876	0.105199722	0.213097157	0.00095182258
2	0.197891869	0.21082041	0.272027876	0.105199722	0.213097157	0.00095182258
3	0.197891869	0.21082041	0.272027876	0.105199722	0.213097157	0.00095182258
4	0.197891869	0.21082041	0.272027876	0.105199722	0.213097157	0.00095182258
5	0.197891869	0.21082041	0.272027876	0.105199722	0.213097157	0.00095182258
6	0.197891869	0.21082041	0.272027876	0.105199722	0.213097157	0.00095182258

Fig. 7. Matrix of steady state probability for six transition states. Source: Authors

It is observed that the probabilities of the states, in each column of Figure 7, are equal (have the same number), that is, they reached their state of stable probability. Table 2 summarizes the steady-state probabilities for each of the six biometric and social states separately. It should be noted that when reaching the state of stable probability it does not mean that the states cannot change or that the system as a whole becomes stationary. In fact, interactions between states continue, but maintaining the same probability.

Table 2 Steady-state probabilities.

State Number	Biometric and Social State	Probabilities
1	Active Military without dependents	0.19789186
2	Marriage	0.21082041
3	Paternity	0.27202787
4	Retirement (with possibility of recall)	0.10519972
5	Definitive Retirement / Disability (Temporary or Not)	0.21309715
6	Resignation	0.00095182

It appears that the ratio between biometric and social states tends to a proportion in which about a tenth of the military is in a situation of retirement (with the possibility of recall). Approximately twice this amount would be in a situation of permanent retirement or disability. It is also noticed that less than a fifth of the military would tend to a situation in which they would remain in active service without dependents.

4 Conclusion

Through the application of Markov Chain techniques, based on a simplified model of the biometric and social states to which a military man may be subject, this paper was able to verify that the states tend to the stable probability. Therefore, it was possible to calculate the steady-state probabilities for each of the aforementioned situations. Considering the available public data, the probabilities were estimated based on the ratio of military personnel in each situation.

The results indicate a trend towards a low proportion of military personnel without dependents. At the same time, for a high proportion of retired military personnel. In the context of the management of the armed forces, for the purposes of governance and internal control, this study serves as a subsidy for military authorities in planning the career flow of their personnel. A brief bibliometric survey on the subject dazzled a relative novelty in this research applied to the military of the Armed Forces, proving their contribution to the academic literature.

The contribution to society can be evidenced by the scope of the study, which directly affects a social protection system with more than one million beneficiaries. In addition, the present study can be replicated to other areas of the public and private sector, which reinforces the importance for society and academia. The present study can be also considered an evolution of Santos (2018), deepening the work of that author, which denotes the relevance of this work for the state of the art.

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