

Vehicle Routing Problem Considering Cargo Theft Probability Applied in a Real Context in the State of Rio de Janeiro: solved using the AIMS software

Bezerra RRR¹, Marchesi JF², Melo ACS³, Nunes DRL⁴, Silva LBP⁵

Abstract The Vehicle Routing Problem (VRP) is a classic well-known combinatorial problem. This paper introduces a factor in the VRP mathematical optimization model considering restrictions that address probability of cargo theft in the regions visited, beyond the traditional constraints such as the number of vehicles, time windows, the capacity of the vehicle and the vehicle's cycle time. The paper proposes a mixed integer linear model that minimizes total transportation costs and cargo theft costs. The model is tested in a real-life case study, a company that distributes pharmaceutical products in Rio de Janeiro. The route solutions with and without cargo theft risk are compared. The model is solved using the AIMMS software.

Keywords: Logistics; Vehicle Routing Problem; Cargo Theft; Mixed Integer Linear Programming; AIMMS.

1 Introduction

The Vehicle Routing Problem (VRP) is a daily problem that affects thousands of companies worldwide, being computed in polynomial time and classified as NP-hard.

The VRP is characterized by defining the best route of delivery of goods from a distribution center to a set of geographically dispersed customers, subject to restrictions (LAPORTE, 2007).

In the literature, there are several studies on VRP. Some focus on the application and development of variations of the classic VRP, as VRP with time windows (Yu, Yang and Yao, 2011), flexible time window (Tas, Jabali and Woensel, 2014), with heterogeneous fleet (Choi and Cha, 2007), with heterogeneous fleet and time window (Jiang et al., 2014), among other combinations. Others seek to improve existing methods of resolution, or develop more efficient methods (not necessarily exact). Some algorithms for the VRP are: ant colony algorithm (Balseiro, Loiseau and Ramonet, 2011), memetic algorithm (Cattaruzza, 2014), neural networks (Torki, Somhon and Enkawa, 1997), among others. Despite the vast literature addressing the VRP, no evidences were found of studies that deal with the VRP considering regions with high probability of cargo theft solved using AIMMS software. Repolho et al. (2019) solved the model VRP with cargo theft using simulated annealing.

¹Rodrigo Rangel Ribeiro Bezerra Example (✉e-mail: rodrigo.bezerra@uepa.br)
Dept. of Engineering. University of the State of Pará. Belém, PA, Brazil.

²Janaina Figueira Marchesi (e-mail: janaina.marchesi@tecgraf.puc-rio.br)
Dept. of Industrial Engineering. Pontifical Catholic University of Rio de Janeiro. Rio de Janeiro, RJ, Brazil.

³André Cristiano Silva Melo (e-mail: acsmelo@yahoo.com.br)
Dept. of Engineering. University of the State of Pará. Belém, PA, Brazil.

⁴Denilson Ricardo de Lucena Nunes (e-mail: denilson.lucena@gmail.com)
Dept. of Engineering. University of the State of Pará. Belém, PA, Brazil.

⁵Leonardo Breno Pessoa da Silva (e-mail: leonardobrenopessoa@hotmail.com)
Post-graduate Program in Industrial Engineering, Federal University of Technology – Paraná, Ponta Grossa, PR, Brazil.

According to the National Association of Cargo Transportation - ANTC (ANTC cited G1, 2014), it is estimated that the value reached R\$ 1 billion in 2013, in Brazil. The Public Security Institute - ISP (2014), shows that there was an increase of more than 84.4% in October 2014 compared to the same period the year before.

The objective of this paper is to solved with AIMMS software a mathematical optimization VRP model, considering restrictions that deal with cargo theft's probability by visited region. The model looks for a routing solution where the first customers to be served are, in general, located in safer areas. The most dangerous areas are served, in general, in the end in order to avoid the theft of consolidated cargo. The cost of theft risk is pondered simultaneously with transport costs.

This paper is organized as follows: section 2 presents the problem formulation; section 3 applied the model to the case study; finally, section 4 describes the main conclusions and derives directions for future studies.

2 Model Formulation

The problem described in this paper is configured as a Vehicles Routing Problem with Time Windows and Heterogeneous fleet, adding the Cargo Theft Probability (Vehicle Routing Problem with Cargo Theft - VRPCT). The model here presented can be generalized to any classical derivation of the VRP, or for any formulation derived from the VRP.

The problem can be defined as follows: given a set of available vehicles (V) with different and known capabilities (Q_v), find the set of routes to serve a given number of customers $|C| \subset |N|$, so as to minimize costs. $|N|$ is a set of nodes that includes customer and the Distribution Center (DC). The indexes i, j e h refer to customers when they assume values between 1 and n , and to the DC when $i = 0$ and $j = n+1$ it, i.e., $N = C \cup \{0, n+1\}$. An additional index v is assigned to the number of vehicles ($v = 1, \dots, |V|$). The vehicles are initially located at the DC. Each customer i demands R_i Stock-Keeping Unit (SKU) from the DC and must be served by exactly one vehicle in a given time window $[A_i, B_i]$, where $B_i \geq A_i$. The time spent at customer i is TA_i . Each customer i is associated a given cargo theft probability, P_i . The distance and travel time between nodes i and j are, respectively, D_{ij} and TD_{ij} , where the latter is measured in function of the former and an average travel speed. CT_v is the transportation cost per km regarding vehicle type v . Additional parameters include the maximum time cycle allowed for each vehicle, TC_v , and the load value L .

The problem includes the following decision variables: (a) arrival time on each client i , is denoted by s_{iv} ; (b) the sequence in which each vehicle performs its itinerary, x_{ij}^v ; (c) the indication of use of the vehicle, y_v ; (d) cargo load on vehicle v right after leaving node i when moving to node j is z_{ij}^v .

The model proposed uses traditional VRP constraints. Constraints (2) to (6) and (11) may be found in Arenales et al. (2007, apud Araujo, 2008), (7), (8), may be found in Kohl et al. (1999), constraint (9) can be found in, Kallehauge (2008). The remaining constraints are adapted and can be found in, Repolho et al. (2019).

$$\text{Minimize } E = \sum_{i=0}^N \sum_{j=1}^{N+1} \sum_{v=1}^V CT_v D_{ij} x_{ij}^v + \sum_{i=0}^N \sum_{j=1}^N \sum_{v=1}^V P_j L z_{ij}^v \quad (1)$$

$$\text{subject to } \sum_{v=1}^V \sum_{j=1}^{N+1} x_{ij}^v = 1, \forall i \in C \quad (2)$$

$$\sum_{j=1}^{N+1} x_{0j}^v = y_v, \forall v \in V \quad (3)$$

$$\sum_{i=0}^N x_{iN+1}^v = y_v, \forall v \in V \quad (4)$$

$$\sum_{i=0}^N x_{ih}^v - \sum_{j=1}^{N+1} x_{hj}^v = 0, \forall h \in C, \forall v \in V \quad (5)$$

$$\sum_{i=0}^N \sum_{j=1}^{N+1} (TD_{ij} + TA_i) x_{ij}^v \leq TC_v y_v, \forall v \in V \quad (6)$$

$$A_i y_v \leq s_{iv}, \forall i \in C, \forall v \in V \quad (7)$$

$$s_{iv} \leq B_i y_v, \forall i \in C, \forall v \in V \quad (8)$$

$$s_{iv} - s_{jv} + (B_i + (TD_{ij} + TA_i) - A_j) x_{ij}^v \leq B_i - A_j, \forall i, j \in C, i \neq j, \forall v \in V \quad (9)$$

$$\sum_{v=1}^V y_v \leq V \quad (10)$$

$$\sum_{i=1}^N \sum_{j=1}^{N+1} R_i x_{ij}^v \leq Q_v y_v, \forall v \in V \quad (11)$$

$$z_{0h}^v \geq \left(\sum_{i=1}^N \sum_{j=1}^{N+1} R_i x_{ij}^v \right) - M(1 - x_{0h}^v), \forall h \in C, \forall v \in V \quad (12)$$

$$z_{hi}^v \leq z_{0h}^v - R_h + M(1 - x_{0h}^v), \forall i, h \in C, i \neq h, \forall v \in V \quad (13)$$

$$z_{ij}^v \leq z_{hi}^v - R_i + M(1 - x_{hi}^v), \forall i, j, h \in C, i \neq j \neq h \neq i, \forall v \in V \quad (14)$$

$$s_{iv} \in R_+, \forall i \in C, \forall v \in V \quad (15)$$

$$x_{ij}^v \in \{0,1\}, \forall i, j \in N, \forall v \in V \quad (16)$$

$$y_v \in \{0,1\}, \forall v \in V \quad (17)$$

$$z_{ij}^v \in Z_+, \forall i, j \in N, \forall v \in V \quad (18)$$

Equation (1) is the objective function, which minimizes total cost given as the sum of travel costs and cargo theft probability costs. Constraint (2) ensures each customer is visited once and one time only by a single vehicle. Constraints (3) and (4) ensures that each vehicle starts and ends the route at the deposit. Constraint (5) is a flow conservation equation that ensures the continuity of each vehicle on the route. Constraint (6) warrants that the cycle time of the vehicle is not exceeded. Constraints (7) and (8) certify that the time windows are respected. Constraint (9) ensures the elimination of sub tours by defining the relationship between the time when a vehicle leaves a customer, the displacement time to the following customer, and consequently the arrival time at the following customer. Constraint (10) ensures that the number of vehicles assigned is smaller than the number of vehicles available. Constraint (11) certifies that vehicle capacity is not exceeded. Constraint (12) calculates vehicle load when it first leaves the deposit. Constraint (13) and (14) track the amount of cargo coming out of each customer. Finally, constraint (15), (16), (17) and (18) define the domain of the decision variables.

3 Case Study

The case study considers a pharmaceutical Distribution Center (DC) located in Rio de Janeiro, Brazil. The DC is responsible for supplying a set of retail pharmacies spread in Rio de Janeiro state. The type of service performed has high frequency of deliveries, low volumes, geographic dispersion of customers and relevant incidence of cargo theft.

This study was conducted in order to define new routing schemes such that the company's costs and possible cargo theft losses are minimized. For the paper's purpose we selected a Day with 14 deliveries (each corresponding to a different retail pharmacy).

The DC has 60 different vehicles with the following capacities (number of SKUs) and costs, respectively, 36 vehicles of 140 and R\$ 0.62/km, 12 vehicles of 180 and R\$ 0.647/km, 6 vehicles of 300 and R\$ 0.734, 3 with a capacity of 450 and R\$ 0.76/km and finally 3 of 600 and R\$ 0.81/km.

The company operates between 8h00 and 17h00, with a 1 one-hour lunch break. Vehicles maximum time cycle is 8 hours. The retail pharmacies can receive goods between 8h30 and 18h. The location of the 14 retail pharmacies was provided by the DC and the distance Origin /Destination (O/D) matrix was determined based on the real road transportation network. The travel time between each $i-j$ pair was then calculated based on the distance O/D matrix and considering an average travel speed of 38 km/h (as informed by the company). The time spent at each retail pharmacy is assumed 6' and 44" (this value was obtained from the company's historical average). Also based on the company's history, we used an average load value for SKU equal to R\$ 387.00.

The probability of cargo theft on each retail pharmacy was calculated considering the records of incidents reported to the police stations in the same area. These data can be found on the Public Security Institute of the State of Rio de Janeiro website. For the purpose, we considered the records between Jan/2012 to Aug/2014. Table 1 summarizes the data input related to demands and theft probabilities.

The main words in all headings (even run-in headings) begin with a capital letter. Articles, conjunctions and prepositions are the only words which should begin with a lower case letter.

Table 1 Pharmacies' demand and theft probability.

Pharmacy	1	2	3	4	5	6	7	8	9	10	11	12	13	14
Demand	24	3	2	12	30	1	18	2	6	27	3	52	223	8
Theft probability (%)	4,2	12,0	8,8	1,7	2,2	19,3	1,7	3,6	19,3	2,2	19,3	2,2	12,0	2,2

The first trials running the model evinced that a great weight was being given to cargo theft costs. Indeed, the model proposed a solution where each retail pharmacy was being served by a vehicle that would transport only its cargo, i.e., 14 vehicles were allocated one to each retail pharmacy. By doing this, the model minimizes the losses with consolidated cargo thefts. However, this solution is not reasonable on a day-to-day operation. In this sense, we ran the model in two steps. First without considering theft probabilities, i.e., $P_i = 0 \quad \forall i \in C$. Second, based on the solution found on Step 1 (the retail pharmacies assigned to each vehicle) the model was ran again considering cargo theft probabilities. The routes executed by each selected vehicle is then optimized regarding the probability of cargo theft. The results obtained in Step 1 and Step 2 are then compared with regard to transportation cost and cargo theft cost.

Running step 1 (no theft probability) 3 vehicles were selected to supply the 14 retail pharmacies. Figure 1 shows the itineraries performed by vehicles 1 and 2 according to Step 1.

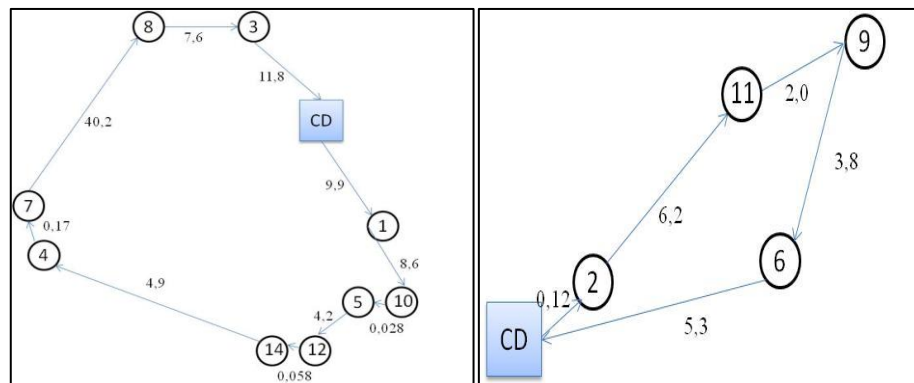


Fig. 1 Step 1 results with no theft probability - (a) Vehicle 1 (b) Vehicle.

Vehicle 1 with a capacity of 180 SKUs serves nine retail pharmacies distributing a total of 175 SKUs. The transportation cost amounts R\$ 56.58. The vehicle executes the following itinerary 0-1-10-5-12-14-4-

7-8-3-0 in 3.31 hours. Vehicle 2 with a capacity of 140 SKUs serves four retail pharmacies distributing a total of 13 SKUs (Table 2). The transportation cost amounts R\$ 10.80. The vehicle executes the following itinerary 0-2-11-9-6-0 in 0.90 hours. Finally, vehicle 3 with a capacity of 300 SKUs distributes 223 SKUs to a single pharmacy (client 13). The transportation cost amounts R\$ 0.43 corresponding to a 0.12 hours route.

In step 2, we consider theft probabilities and optimize the routes obtained in step 1. As vehicle 3 is serving only one pharmacy (number 13), the solution remains. Note that pharmacy 13 demands more (223 SKUs) than all the other pharmacies together (188 SKUs). Figure 2 shows the itineraries performed by vehicles 1 and 2 according to Step 2.

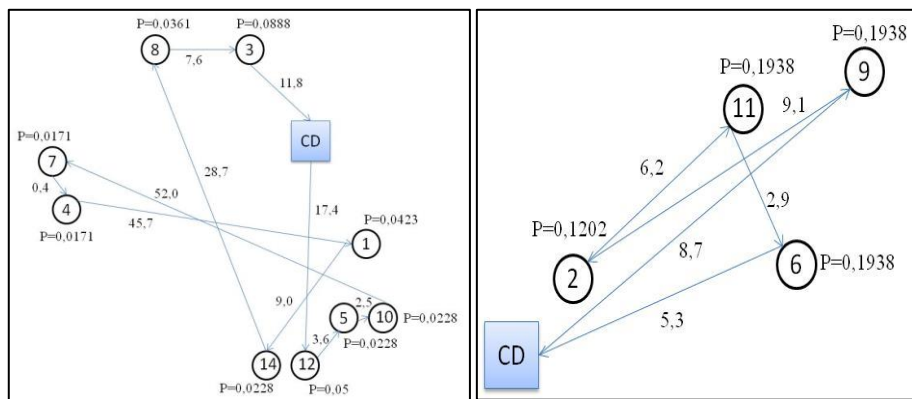


Fig. 2 Step 2 results with theft probability - (a) Vehicle 1 (b) Vehicle 2.

With regard to the routes performed by vehicles 1 and 2, they were optimized separately by fixing the vehicle and the pharmacies to serve. The results are as follows. Vehicle 1 serves now the nine pharmacies with a transportation cost of R\$ 115.62 and performing the following itinerary 0-12-5-10-7-4-1-14-8-3-0 in 5.43 hours. Vehicle 2 serves now the four pharmacies with a transportation cost of R\$ 19.96 and performing the following itinerary 0-9-2-11-6-0 in 1.29 hours. Vehicle 2 route has a curious result. Despite pharmacy 9 has a higher theft probability than pharmacy 2 and it is located farther, the former is served first as its demand is twice bigger than the one of pharmacy 2.

Table 2 summarizes the results obtained for each vehicle in Step 1 and Step 2 with regard to cargo theft costs and cargo load on each vehicle after leaving each node (z_{ij}^v). With regard to vehicle 1, transportation costs are 104% higher and cargo theft costs amount R\$ 5,028.22, while the respective solution in Step 1 amounts R\$ 6,988.00, i.e. 22% less. With regard to vehicle 2, transportation costs are 85% higher and cargo theft costs amount R\$ 1,675.66 instead of R\$ 1,954.87, i.e. 14% less.

As expected, transportation costs are now higher as routes optimize the cargo arriving at each pharmacy with regard to the probability of theft rather than just transportation costs. Consequently, time cycles are also higher, though respecting the maximum time cycle allowed for each vehicle.

The solutions obtained give the decision maker the ability to make choices regarding daily routes based on the probability of thefts and the company aversion to risk. A company willing to take risks may opt by the solution provided by traditional VRP models, while a more careful one may opt by the VRPCT model.

Table 2 Comparison Cargo Theft.

Vehicle (capacity)	Route without probability	z_{ij}^v (SKUs)	Cargo theft cost (R\$)	Route probability	with z_{ij}^v (SKUs)	Cargo theft cost (R\$)	
1 (1800 SKUs)	1	175		0	175		
	1	151	2,862.43	12	123	1,545.71	
	10	124	1,333.73	5	93	1,086.41	
	5	94	1,095.25	10	66	821.43	
	12	42	830.27	7	48	437.22	
	14	34	370.97	4	36	317.97	
	4	22	225.23	1	12	588.84	
	7	4	145.74	14	4	105.99	
	8	2	55.94	8	2	55.94	
	3	0	68.70	3	0	68.70	
	0			0			
		Cost/Route	6,988.25			Cost/Route	5,028.22
2 (1400 SKUs)	2	13		0	13		
	2	10	604.95	9	7	974.94	
	11	7	749.96	2	4	325.74	
	9	1	524.97	11	1	299.98	
	6	0	75.00	6	0	75.00	
	0			0			
		Cost/Route	1,954.87			Cost/Route	1,675.66

3 Conclusion

This paper proposes a solved using AIMMS software for the vehicle routing problem where risk of cargo theft is taken into account. The recent model minimizes transportation costs and theft costs. The solutions proposed are apparently less obviously from a geographic perspective but allow the reduction of cargo transported to areas that are more dangerous. This problem is especially relevant in South American countries where the number of cargo thefts is highly relevant. The solutions obtained show that the model is relevant, though needing some calibration in the way theft costs are summed to transportation costs. Indeed, considering the full cargo value produces distorted results as, in general, transportation costs are much smaller than the former ones. For that reason in the case study presented, the model was solved in two steps. Future work includes solving this issue, by considering the actual costs for the companies taking into account insurance coverage. It is intended to test heuristics to solve any problems. Additionally, a new approach where theft is considered in the arches and not in the nodes is being thought. As for the specific case of Brazil, it is intended to study the effect that some security policies, such as the Pacifying Police Units (UPP), might have on this problem

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