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Improving the performance of the attribute chart np_x

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Abstract. This paper proposes an improvement in terms of speed of detecting anomaly/changes in the attribute np_x chart employing variable sample size for monitoring the mean value of a variable (X) in a process. Sequential sample size of n_a, n_b are extracted with $n_a > n_b$. Each item is classified as approved or not according to a discriminant limit (z). In the end we'll have Y_a and Y_b classified as disapproved items. Whenever $Y_a > UCL_{na}$ or $Y_b > UCL_{nb}$, the process is judged to be out of control and after adjustments in the process, the inspection is always restarted with a sample size n_a. The parameters used in the construction of the np_x chart with simplified variable sample size were obtained through a search for values that optimize their performance, such that can compete with the traditional $\bar{X}$ control chart. The performances were compared with traditional $\bar{X}$ control chart in terms of average run length *(ARL)* in scenarios of shift (δ) in the process mean.

Keywords: Control Chart, Variable and Attribute Inspection, Monitoring the Process Mean, Variable Sample Size, Average Run Length.;

1 Introduction

The compliance of the specifications from a process is often monitored with the aid of control charts, being the traditional Shewhart $\bar{X}$ control chart widely used, due to its good performance and operational simplicity (Costa, 1994). The use of attribute control charts for monitoring the mean provides numerous advantages in terms of cost, time and simplicity, especially when we are dealing with destructive tests.

However, several studies have been developed to propose new monitoring strategies that can provide better performing charts. Wu et al. (2009) proposed a control chart based on attributes (items are classified as approved or not according to the warning limits), for the purpose of monitoring the mean process. The procedure consists to classify each item of a fixed sample size n as approved or not according to the warning limits w using a typical gauge "Go/No Go". Fig. 1 shows an example of this type of tool. Let X_i, the value of a quality characteristic of the $i\text{-}th$ sampled item and w_u, the warning limit. If $X_i > w_u$, then the item is classified as disapproved; otherwise, as approved. It is important to mention that when the item is classified as disapproved according to the warning limit, it does not mean the item is inappropriate, non-conforming but due to only its characteristic value is higher than w_u. Let Y be the number of items classified as disapproved. If $Y > UCL$ then the process is judged to be out of control. Wu et al (2009) observed that np_x control chart (to monitor a process mean) will have a similar performance of the traditional $\bar{X}$ if we double the sample size used for $\bar{X}$ chart. Such argument is the key as an attribute inspection is faster, cheaper and completely feasible. In the traditional np control chart, an item is classified as conforming if the inspected item satisfies a set of requirements stated by the engineering team like the specification limits; otherwise, the item is classified as non-conforming.

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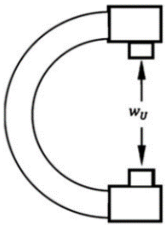

Fig. 1. Ring Gauge (Source: Sampaio, E. S., Ho, L. L., and Medeiros, P. G (2013) A Combined $np_x - \bar{X}$ Control Chart to Monitor the Process Mean in a Two-Stage Sampling. John Wiley & Sons)

Furthermore, Prabhu, Runger and Keats (1993) performed a study in which a double sample sizes scheme was proposed, keeping the sampling interval fixed, but without necessarily restricting the sample size. In this study $\bar{X}$ control chart was used with adaptive sample size.

Costa (1994) developed a study to compare in terms of speed of detecting changes in the mean process employing variable sample sizes (VSS), variable sampling intervals (VSI) and double sampling (DS) on the traditional Shewhart $\bar{X}$ control chart.

Zhou and Lian (2010) proposed an attribute control chart with adaptive sample size, with inspection level categories according to the status of the process and compared its performance against that of a simple sampling (traditional) np control chart. However, it is possible to realize a certain complexity in managing the inspection scheme and making operator's decisions.

Sampaio, Ho and Medeiros (2013) proposed a scheme that employs two sampling stages which combines two control charts to monitor the mean of a process: an attribute control chart and the variable traditional Shewhart $\bar{X}$ control chart.

The purpose of this article is to propose improvements to np_x control chart (Wu et al., 2009) with the application of a simplified scheme of variable sample size for monitoring the mean of a quality characteristic of interest (X) such that, provides a performance able to compete with the traditional $\bar{X}$ control chart.

In this study some assumptions are assumed as the quality characteristic of interest follows a Normal distribution, with known mean μ_0 and standard deviation σ_0, (it remains unchanged), and independent observations are collected. Due special causes, the mean value μ_0 may change to $\mu_1 = \mu_0 + \delta\sigma_o$, where δ expresses the magnitude of the shift in terms of standard deviation (σ_o). After this introductory section, the simplified variable sample size scheme applied to np_x control chart is presented in Section 2; in Section 3 its performance is compared with the traditional $\bar{X}$ control chart and Section 4 provides the conclusions of this study.

2 Simplified Variable Sample Size Procedure of np_x control chart

For an evaluation by attributes it is common to use a ring gauge ("Go / No Go") to check whether the discriminant/warning limits have been exceeded or not (Kennedy et al., 1987). In the construction of attribute control chart np_x with simplified variable sample size for monitoring the mean process, sequential samples of sizes n_a and n_b ($n_a > n_b$) are extracted. Let X_{ki} be the $i-th$ item from the sample of size n_k, $k = a, b$. If $\frac{X_{ki} - \mu_0}{\sigma_0} > z_k$ the item is classified as disapproved; otherwise, as approved with $z_k = \frac{W_k - \mu_0}{\sigma_0}$, the standardized discriminants limits, (W_k, the non-standard discriminants limits). At the end of the inspection we will have Y_k disapproved items, $k = a, b$.

The inspection procedure consists of the following steps:

1. The monitoring begins by taking a sample of size n_a and obtains Y_a using a ring gauge for example. If $Y_a < UCL_{na}$ then the process is judged to be in control and go to step 2, otherwise, the process is judged to be out of control (a search for the special causes begins) and go to the step 3;

2. Take a sample of size n_b and obtain Y_b . If $Y_b < UCL_{nb}$ then the process is judged to be in control, otherwise, the process is judged to be out of control (and a search for the special causes begins) and go to step 3;
3. Repeat the step 1.

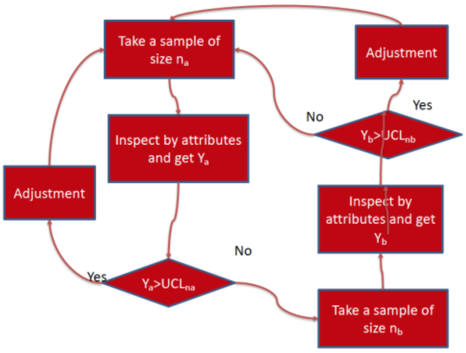

Fig. 2. Inspection procedure of the simplified variable sample size for np_x control chart. Source: Authors

Figure 2 illustrates the inspection procedure. Figure 3 shows an example of the simplified variable sample size for np_x control chart. Observe that the inspection at the 5th sample (blue square) presents $Y_a > UCL_{na}$ then the process is halted for adjustment, the inspection procedure is restarted in the 6th sample by using a sample size n_a, following by a sample size n_b. Note that the 9th sample (purple diamond) presents $Y_b > UCL_{nb}$ as in the previous case the process is halted for adjustment.

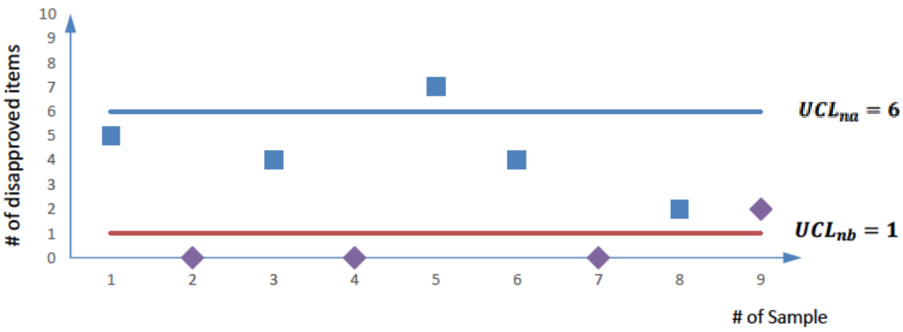

Fig. 3. Simplified Variable Sample Size for np_xcontrol chart. Source: Authors

The procedure described above is applied for cases of increases in the mean process ($\Delta > 0$), being $\Delta = \mu_1 - \mu_0$ and $\mu_1 = \mu_0 + \delta\sigma_0$. For $\Delta < 0$ or $\Delta \neq 0$ few adjustments of the described procedure are necessary.

The inspection scheme presented can be described through a Markov chain with the following transition states:

- State A, Y_a is within control limits
- State B, Y_b is within control limits
- State C, Y_b is out of control limits
- State D, Y_a is out of control limits

The transition matrix that describes this Markov chain can be written in the following matrix $\boldsymbol{P}$:

$$\boldsymbol{P} = \begin{matrix} \\ A \\ B \\ C \\ D \end{matrix} \begin{matrix} \begin{matrix} A & B & C & D \end{matrix} \\ \begin{bmatrix} 0 & PB & 1-PB & 0 \\ PA & 0 & 0 & 1-PA \\ PA & 0 & 0 & 1-PA \\ PA & 0 & 0 & 1-PA \end{bmatrix} \end{matrix} \quad (1)$$

Where:
- PA refers to the probability of being in State A
- PB refers to the probability of being in State B

The probabilities PA and PB are calculated as follows:

$$PA = \sum_{Y_a=0}^{UCL_{na}} \binom{n_a}{Y_a} p_{0a}{}^{Y_a} (1 - p_{0a})^{n_a - Y_a} \quad (2)$$

$$PB = \sum_{Y_b=0}^{UCL_{nb}} \binom{n_b}{Y_b} p_{0b}{}^{Y_b} (1 - p_{0b})^{n_b - Y_b} \quad (3)$$

The values of p_{0k} in (2) and (3) are the probabilities of an item to be disapproved, expressed by $p_{0k} = \mathbb{P}\left(\frac{X_{ki} - \mu_0}{\sigma_0} > z_k\right), k = a, b.$

The transition matrix $\boldsymbol{P}$ is irreducible, aperiodic, and $\boldsymbol{P^t}$ converges to a matrix in which all rows are equal to the vector $\boldsymbol{\pi}$ when $\boldsymbol{t} \to \infty$. The vector $\boldsymbol{\pi} = (\pi_1, \pi_2, \pi_3, \pi_4)$ is the stationary distribution and each element π_i, $i = 1, 2, 3, 4$ is associated with one state of $\boldsymbol{P}$, and the vector π can be obtained as the solution of the linear system of equations $\boldsymbol{\pi} = \boldsymbol{\pi} \times \boldsymbol{P}$, subject to the restriction $\sum_{i=1}^{4} \pi_i = 1$. By solving the mentioned system, $\pi_1 = \frac{PA}{1+PA}$; $\pi_2 = \frac{PA \times PB}{1+PA}$; $\pi_3 = \frac{PA - PA \times PB}{1+PA}$ and $\pi_4 = \frac{1-PA}{1+PA}$. The values π_3 and π_4 indicate the long-term probability of the inspection signaling that the process is out of control, using the samples n_a and n_b, respectively. The average sample size used in this procedure is expressed as $ASS = n_a(\pi_1 + \pi_4) + n_b(\pi_2 + \pi_3)$.

A metric that has been adopted by many authors to evaluate the performance of the control charts is the average run length *(ARL)*, which expresses the number of samples until an indication of an out of control condition (Montgomery, 1985). Thus, the average number of samples up to the signal of an out of control condition, considering the matrix $\boldsymbol{P}$ above mentioned can be expressed by $ARL_1 = \frac{1}{\pi_3 + \pi_4}$. When the process is in control, we will have:

$$ARL_0 = \frac{1+PA}{1-PA \times PB} \quad (4)$$

The parameters of control chart of the current proposal are: the sample sizes n_a and n_b, the control chart limits for each sample size UCL_{na} and UCL_{nb}, the standardized discriminant limits z_a and z_b. After chosen the sample sizes n_a and n_b, the control chart limits UCL_{na} and UCL_{nb}, the standardized discriminant limits z_a and z_b are determined by an intensive search as follows:

```
For z_a=0 to 3 by 0.005;
    For z_b=0 to 3 by 0.005
        For UCL_na=0 to n_a by 1
            For UCL_nb=0 to n_b by 1
            Calculate ARL_0c by expression (4),
            If |ARL_0c − ARL_0T|<1 then
            Save z_a, z_b, UCL_na, UCL_nb
            End
        End
    End
End
```

ARL_{0T} is the target value of ARL_0 like 370.

3 The performance of the simplified variable sample size of np_x control chart

In this section the performance of the simplified variable sample size of np_x control chart is described. The control limits and the discriminant limits are set to get ARL_0 closer to 370. We presented 2 cases: the parameters of the first case are: $n_a = 11; n_b = 2, z_a = 0.720; z_b = 2.400, UCL_{na} = 6; UCL_{nb} = 1$. With these parameters the matrix $\boldsymbol{P}$ is equal to:

$$\boldsymbol{P} = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0.99993 & 0.00007 & 0 \\ 0.99466 & 0 & 0 & 0.00534 \\ 0.99466 & 0 & 0 & 0.00534 \\ 0.99466 & 0 & 0 & 0.00534 \end{bmatrix} \end{array} \quad (5)$$

which yields $\pi_1 = 0.49866$, $\pi_2 = 0.49863$, $\pi_3 = 0.00003$, $\pi_4 = 0.00268$, values of $ASS = 6.512$ and $ARL_0 = 369.075$.

For the second case the same sample sizes of first case is used but the discriminant and control limits are set at: $z_a = 0.500; z_b = 2.195, UCL_{na} = 7; UCL_{nb} = 1$

$$\boldsymbol{P} = \begin{array}{c} \\ A \\ B \\ C \\ D \end{array} \begin{array}{c} \begin{array}{cccc} A & B & C & D \end{array} \\ \begin{bmatrix} 0 & 0.99980 & 0.00020 & 0 \\ 0.99479 & 0 & 0 & 0.00521 \\ 0.99479 & 0 & 0 & 0.00521 \\ 0.99479 & 0 & 0 & 0.00521 \end{bmatrix} \end{array} \quad (6)$$

which yields $\pi_1 = 0.49869, \pi_2 = 0.49860, \pi_3 = 0.00010, \pi_4 = 0.00261$, values of $ASS = 6.512$ and $ARL_0 = 369.051$.

In Table 1, values of ARL_1 are obtained considering shift sizes of $\delta = \{0.25, 0.50, 0.75, 1.00, 1.25, 1.50, 1.75, 2.00, 2.25, 2.50, 2.75, 3.00\}$ (see last two columns). Note that the performances of these two cases are very similar. For comparative purposes, the values of ARL_1 of traditional control charts $\bar{X}$ (using a fixed sample size equal 6) and np_x (a fixed sample size equal 12 - double of six) are also put together in the same table (second and third columns).

According to Table 1, we can see that the simplified variable sample size procedure for np_x control chart provokes a reasonable improvement. It can compete with $\bar{X}$ control chart using an average sample size slightly larger than the used for $\bar{X}$ chart as the ARL_1 values of the current proposal are similar of the $\bar{X}$ control chart for all shift sizes considered in the current study.

Table 1. ARL_1 values of the control charts: traditional $\bar{X}$ control chart, traditional np_x control chart and the current proposal.

δ	$\bar{X}$ control chart	np_x fixed sample size	np_x simplified variable sample size	
0.25	66.614	66.621	64.073	64.507
0.50	16.754	16.219	16.128	16.395
0.75	5.803	5.406	5.590	5.744
1.00	2.705	2.461	2.552	2.634
1.25	1.639	1.496	1.513	1.566
1.50	1.229	1.151	1.146	1.173
1.75	1.071	1.036	1.033	1.043
2.00	1.017	1.006	1.005	1.008
2.25	1.003	1.001	1.001	1.001
2.50	1.000	1.000	1.000	1.000
2.75	1.000	1.000	1.000	1.000
3.00	1.000	1.000	1.000	1.000
n	$n_{\bar{X}}$ = 6	n_{npx} = 12	n_a = 11 n_b = 2	n_a = 11 n_b = 2
UCL	$UCL_{\bar{X}}$ = 1.136	UCL_{np} = 3	UCL_{na} = 6 UCL_{nb} = 1	UCL_{na} = 7 UCL_{nb} = 1
$discriminant$ $limits$	-	w_u = 1.620	z_a = 0.720 z_b = 2.400	z_a = 0.500 z_b = 2.195

4 Conclusions

Many strategies for monitoring processes that result in better performance of control charts, applied in diverse areas have been proposed. In this study an improvement for the attribute chart np_x with simplified variable sample size scheme to monitor the mean process is proposed.

These preliminary results emphasized the potentiality of the simplified variable sample size procedure for np_x control chart. The proposal proved to be very competitive as it presented similar performance (in terms of ARL_1) of the traditional $\bar{X}$ control chart in all shift sizes employing an average sample size slightly higher than the used for $\bar{X}$ control chart. It is relevant to emphasize that no measurement is made on the sampled units. They are classified as disapproved or approved using a ring gauge, for example.

Therefore, these promising results inspire the authors to deepen on this subject exploring other aspects as: how are the impacts if the same discriminating limit is used to classify n_a and n_b items, which should be the sample sizes of n_a and n_b that are able to compete with the $\bar{X}$ control chart, to list a few.

Through this study it was possible to conclude that with an adequate selection of parameters in the classification of items, we can have an early signaling using control chart by attributes with simplified variable sample size, which competes with the $\bar{X}$ control chart, providing a superior signaling (in other words, signaling in advance). The current proposal can be viewed as a good alternative to monitor changes in the mean process, in view of potential advantages in terms of cost, time, simplicity, and/or in especially scenarios where we deal with destructive tests, requiring less expertise to perform the monitoring, in addition it doesn't require complex measuring instruments to support decision making during the evaluation of classified items.

This study can be also complemented in the future for inspections by attributes, considering EWMA scheme for the monitor the mean value of a variable (X), with few adjustments used in this paper.

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