Operational Research as a Tool for the Selection of Mortality Tables: The Actuarial Problem of the Brazilian Armed Forces

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Abstract The military of the Armed Forces are not subject to any social security regime, being supported by a specific social protection system, supported by the Union's budget forecast. For the correct measurement of costs with the veteran military and pensioners, it is necessary continuous study and monitoring of its actuarial forecasts. These, in turn, require the appropriate selection of the actuarial tables, to represent the sample of the population under analysis as faithfully as possible. The objective of this paper is to determine, among the main Goodness-of-Fit (GoF) tests disseminated in the academia, which is the best one to be applied for the selection of actuarial tables for use in the data of the military of the Brazilian Armed Forces. Consequently, making it possible to select the actuarial tables that present the best GoF for that population sample. The results indicate that the most favorable test for the studied population is the Kolmogorov-Smirnov test.

Keywords: Actuarial Table Selection; Goodness-of-Fit Tests; Social Security; Armed Forces.

1 Introduction

Populations in industrialized countries are aging rapidly and this structural change puts pressure on public finances, social support programs that target retired persons, and the sustainability of pension systems (Böckerman and Ilmakunnas, 2019). This has created the need for researchers to better understand the underlying patterns of mortality improvement, and use these patterns to project mortality credibly into the future (Chang and Shi, 2020).

In the last years, rapidly aging populations and increased longevity in many developed countries have been a growing concern for governments and societies (Shang, 2020). These concerns are focused on the sustainability of social security, health and aged-care systems, especially due to the increased longevity among populations (Shang, 2020). The increase in longevity, the ultra-low interest rates and the guarantees associated with pension benefits have put significant strain on the pension industry (Boado-Penas *et al.*, 2020). The increase in life expectancy over the past several decades has been impressive and represents a key challenge for institutions that provide life insurance products (Arnold *et al.*, 2019).

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Aiming to increase public governance, the Federal Court of Accounts (FCA) urged the Brazilian Ministry of Defense (MoD), from 2010, to carry out annually a projection of actuarial costs with pensioners of the Armed Forces (AF), considering a time horizon of 75 years. This task proved to be a great organizational challenge, due to the large volume of data, the computational effort required by these mathematical calculations and uncertainties involved (Gomes; Santos; Martins, 2017). In these cases, the Operational Research (OR) presents great relevance, because it makes use of mathematical and/or logical models to solve real problems, presenting a highly multidisciplinary (Costa *et al.*, 2020; Santos *et al.*, 2015).

Actuarial Calculation applies scientific and technical principles of many other subjects, including mathematics, probability theory and statistics, to problems involving risk, uncertainty and finance (Bühlmann, 1997; Ogborn, 1956). Actuarial projections of income and expenses are important to ensure actuarial balance and reduce the risk of lack of liquidity, enabling public agencies to provide resources for subsequent years (Gomes *et al.*, 2017). The purpose of these projections for a pension and social security entity, therefore, would be to quantify the estimated future costs with payment of benefits and the estimated future revenues of contributions from participants. According to Tenório *et al.* (2020), the AF has been facing successive budget restrictions, which requires resources to be used as efficiently as possible.

For the correct projection of actuarial costs, on the other hand, it is necessary to determine the actuarial tables of mortality and disability that best represent the expectations of decrements, that is, death and disability of the population studied. This article aims to determine, among the main Goodness-of-Fit (GoF) tests disseminated in the academia, which test best applies for the selection of actuarial table for use in the data of the Brazilian AF. Secondly and consequently, select, from the application of GoF tests, which is the actuarial table that best represents the particularity of AF staff. In this paper, the calculation was limited to the best mortality table for the group of active, inactive and pensioner militaries.

2 Background

Actuarial Tables are tables containing biometric and social characteristics of a certain population for risk analysis and expectations in Actuarial Sciences. The tables selected in an actuarial study must effectively represent the biometric events (death, disability, disease, etc.) to which the analyzed population is submitted, and should be chosen based on historical experience and the perspectives of the sample. Most studies of seasonal variation in mortality rely on aggregated death counts at the population level. The use of actuarial tables mismatched from reality can result in cumulative actuarial losses or gains over time, creating structural imbalances to the system (Teixeira, 2020).

The actuarial table is the basic tool for analyzing population change, being the oldest demographic model in use, having been used historically to measure the longevity of a population (Castro, 1997). The earliest known tables date back to the 3rd century, but modern actuarial ones were developed from the 17th century. Since then, its practical applications have been diversified, with new relationships developed and functions improved. The Actuarial Tables are the foundation for any product in the social security area and may come from data from demographic censuses or the insurance companies' experience (Santos, 2018).

GoF tests are statistical methods that allow to examine how well a sample of data agrees with a certain distribution, as its population. The objective of the GoF tests is to evaluate whether two frequency distributions are approximately identical or if they are considered heterogeneous (D'Agostino and Stephens, 1986).

Although there is no single internationally standardized definition for the GoF tests, the Glossary of Statistics presents a general concept about the English term Goodness-of-Fit (Assis, Janilson Pinheiro de; Souza, Roberto Pequeno de; Dias, Santos, 2019): "Statistical test of probability distribution model, in which the observed proportions are adjusted to the expected proportions, mathematically deduced or established according to some theory. It is also called Goodness-of-Fit" (Assis; Dias, Santos, 2019).

GoF tests consist of verifying the suitability of a probabilistic model to a data set. In the GoF tests, there is a null hypothesis H_0 that X, a random variable, follows a declared probability law F(x). The techniques of these tests consist of mathematical models to measure the conformity of the data of a sample, that is, set of values of x with the hypothetical distribution; or, equivalently, with its discrepancy about it. In other words, the basic concept is that, given a random sample of size n, observed from a random variable X, it is desired to test the null hypothesis H_0 that the sample follows a certain distribution function F(x), confronting it with the alternative hypothesis H_1 that the sample does not follow the distribution function F(x):

H_0: X has distribution F (x) vs H_1: X has no distribution F(x).

In the formal structure of the adhesion test, the null hypothesis H0 can be a simple hypothesis, when F(x) is specified completely or H0 can provide an incomplete specification and then it will be a compound hypothesis. Among the most used GoF tests in actuarial studies and the social security market, the Qui-square, Kolmogorov-Smirnov (KS) and Quadratic Measure Deviation (QMD) tests were selected (Teixeira, 2020).

2.1 Qui-square test

To test whether the calculated discrepancies have statistical significance, the index x^2 is calculated and compared to the same factor ($x^2_{critical}$), obtained from the qui-square distribution table (Santos, 2018). The test statistic can be calculated by (1):

$$x^2 = \sum_{i=1}^{n} \frac{(O_i - E_i)^2}{E_i} \quad (1)$$

Where x^2 = Qui-square Test Statistics; O = Observed Frequency; E_i = Expected frequency.

The result x^2 expresses the divergence between the observed frequency and the expected frequency and, the lower this value, the greater the probability of not rejecting the hypothesis of GoF. Once the variable x^2 is calculated, it is compared to $x^2_{critical}$, considering the level of arbitrary significance and the degrees of freedom considered in the test (Barceló, 2018).

2.2 Kolmogorov Smirnov (KS) test

The KS test statistic is calculated from the supreme value, that is, the maximum of the values in a certain interval (Kolmogorov, 1933) (2).

$$D_n = sup|F_n(x) - F_0(x)| \quad (2)$$

Where F_n (x) and F_0 (x) represent the cumulative distribution functions of expected and observed deaths, respectively. The statistic of test D_n will be the greatest vertical distance between F_n (x) and F_0 (x) that is (3):

$$D_n = max\{|D_n^-|, |D_n^+|\} \quad (3)$$

Where $D_n^- = F(x_{k-1}) - F_0(x_k);\ and\ D_n^+ = F(x_k) - F_0(x_k)$.

The KSz statistic can be calculated by (4):

$$KS_Z = \sqrt{n} D_n \quad (4)$$

The critical value of the statistical distribution of KS is compared with the tabled and H_0 will be rejected if $D > D_{tabled}$ (α).

2.3 Quadratic Measure Deviation (QMD)

Given a series of measures of magnitude, the differences between the measured values and the most likely value, $\overline{\chi}$, are called deviations or residues (Armstrong and Collopy, 1992). A way to measure dispersion could be the quadratic measure deviation, defined by (5):

$$S_{RMS} = \sqrt{\frac{\sum_{i=1}^{n}(\delta x_i)^2}{n}} \qquad (5)$$

Where S_{RMS} is the Quadratic Measure Deviation in the event occurrence; δx_i is the difference between the expected and observed probabilities of the event.

3 Selection of Actuarial Tables

A conceptual model for the selection of Actuarial tables, called Adherence Cube, represented by a three-dimensional matrix that contains the original, aggravated and unburdened probabilities of death/disability was proposed by Gomes, Santos and Martins (2017). The first dimension of this cube indicates the actuarial tables of death or disability and the second dimension corresponds to the ages of the participants (Gomes; Santos; Martins, 2017). The third dimension refers to 198 variations, representing 99 aggravations and 99 distinct percentage reductions. In addition to the aggravations and reductions percentages already explained, a fourth dimension was inserted, composed of aggravations and reductions in years of life, in the ages of the actuarial tables, was proposed the use of the "Adherence Hypercube" (Teixeira, 2020).

The application of GoF tests can be performed in two ways: from the Exact Age Calculation or to Age Classes. In the first, the test is performed for the entire population sample, considering all ages of the population. In the GoF test for age class, the sample is divided by age groups and each class represents an independent observation of the sample. In this case, the test is performed for each class and integrated to present the result that best suited that population.

It is worth mentioning that as the AF databases are quite large, historically, only the GoF test for Exact Age and not by Age Classes is used. In the present study, the two forms of testing were applied. In this paper, data from the Strategic and Managerial Information Bank (SMIB), administered by the MoD, were used. The information was consolidated in 2017 for the year 2015, referring to the Brazilian Navy (BN), Brazilian Army (BA) and Brazilian Air Force (BAF). Thus, as illustrated in Table 1, more than 1.5 million historical records were analyzed. This database includes information from the military regarding dates of birth, entry into the AF, disability, death and others, whose quantities are subdivided by active and inactive military personnel, pensioners and beneficiaries.

Table 1. Quantitative of active/inactive militaries and pensioners in 2015.

	BN	BA	BAF	AF
Active/inactive	212.580	951.941	229.172	1.393.693
Pensioners	47.985	101.569	34.652	184.206
Total	260.565	1.053.510	263.824	1.577.899

Source: Ministry of Defense (2017).

It is worth mentioning that, in this article, a sample of data for the year 2015, although not the most recent, meets the need for research. This data volume was chosen because it was treated at the Naval Systems Analysis Centre (CASNAV) and considered more robust, consistent and coherent than other samples; moreover, because it was the same mass of data used by Santos (2018), a pioneer in actuarial modeling for AF.

The actuarial tables of mortality used in this study were: ALLG-72; American Experience; AT-49; AT-50; AT-55; AT-71; AT-83; AT-83 Male; AT-2000; AT-2000 F; BR-EMSSb-v.2010-m; BR-EMSSb-v.2010-f; BR-EMSmt-v.2010-m; BR-EMSmt-v.2010-f; BR-EMSSb-2015-m; BR-EMSSb - 2015 - f; BR-EMSmt-2015-m; BR-EMSmt-2015-f; CSO2001MALE; CSO2001FEMALE; CSO-41; CSO-58; CSO-80;

CSG-60; GAM-1971; GAM 1994 Male; GKM-70; GKM-80; GRM-80; GRF-80; GRM-95; GRF-95; American Group; Semitropical Hunter; IAPC; IBGE-2011-M; IBGE-2011-F; IBGE-2011; IBGE-2012-M; IBGE-2012-F; IBGE-2012; Prudential 1950; SGB-51; Rentiers Français ; RP-2000 - 1992 Base-Male Aggregate; SGB-71; SGB-75; USTP-61; UP-84; UP94Men; UP94Women; UP-94 MT-M-ANB; and X-17. These tables are commercial and public, so their specificities can be found on the website of the Brazilian Institute of Actuary (BIA) and on the website of the Society of Actuaries (SOA).

3.1 Application of GoF tests

In the present article will be used the Qui-square, KS and QMD tests. For the application of GoF tests, the commercial software "Aderência 4D", developed by the Arcadia Systems Company, was used. The 4D Grip software uses the most modern techniques of Software engineering and, through the resources of Parallel Computing, presents the results of the tests of GoF in minimal computational time.

3.1.1 Exact Age Calculation Tests

For the population of active, inactive and pensioners of the AF, GoF tests were performed for two possible entries. The first entry of mortality data considers the entire population, that is, all ages; while the second data entry considers a representative sample of this population, with the age range between 20 and 90 years, which comprises 89% of the population data.

Considering the first entry, with the entire population, there was no actuarial table that had GoF to the observed mortality data, based on the Qui-Square test. Using the Kolmogorov-Smirnov test, there was GoF of 481 tables. The table with the best adhesion was the AT-83 Male decreased by age in 3 years, according to Fig. 1.

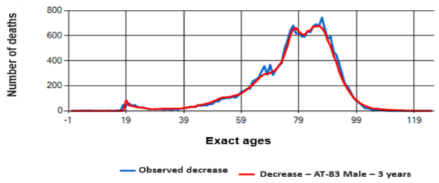

Fig. 1. KS Test: The best GoF for all ages.

From the QMD test, the table with the best GoF was the CSO2001FEMALE table, which was reduced by 5%, according to Fig. 2.

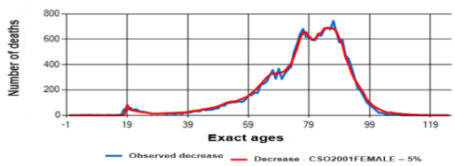

Fig. 2. Best GoF using QMD Test for all ages.

For the second entry, with the age range between 20 and 90 years, a single table adhered to the mortality observed from the Qui-Square test: GKM-70 reduced by 61%. Using the Kolmogorov-Smirnov test, there was GoF of 1.071 tables, and the best one was verified using the BR-EMSSb-2015-M table, aggravated by 7%. From the QMD test, the table with the best GoF for the second entry aged between 20 and 90 years was the CSO20001FEMALE, reduced by 5%. Table 2 summarizes the comparison of the number of tables adhered to in each test.

Table 2. Number of General Mortality Tables adhered from each test.

Data \ Test	Qui-square	KS	QMD
The Entire Population	There was no GoF	481	12.667
Sample aged between 20 and 90 years	1	1.071	12.667

Table 3 highlights the table that presented the best GoF in the calculation for the exact age.

Table 3. General Mortality Tables with the best GoF from each test.

Data / Test	Qui-square	KS	QMD
The Entire Population	There was no GoF	AT-83 Male reduced by age in 3 years	CSO 2001FEMALE decreased by 5%
Sample aged between 20 and 90 years	GKM-70 reduced by 61%	BR-EMSSb-2015-M increased by 7%,	CSO 2001FEMALE decreased by 5%

3.1.2 Tests by Age Classes

Considering the first entry, with the entire population, the lowest class was established in the lower age range 0 (zero) years and age above 19 (nineteen) years. Seven intermediate classes with a ten-year amplitude were established. The upper class was established with a lower age of 90 (ninety) years and upper age of class 125 (one hundred and twenty-five) years. For this entry, there was no actuarial table that adhered to the observed mortality data from the Qui-Square test. With the KS test, there was GoF of 1.203 tables. The table with the best GoF was CSO 2001FEMALE, which was reduced by 6%.

For this same entry, considering the entire population, with the same age class division, through the QMD test, the table with the best GoF was the CSO 2001 FEMALE decreased by 7%. For the second entry, with the interval of ages between 20 and 90 years, using the Qui-Square test, there was still no actuarial table that had GoF to the observed mortality data. Using the KS test, there was GoF of 1.936 tables. The lower class of the sample was considered to be included in the age range below twenty years and older than thirty years old. Five intermediate classes with a ten-year amplitude were established. The upper class was established with a range between eighty and eighty-nine years old. For this age class division, the table with the best GoF was AT-83 reduced by 21%, as illustrated in Fig. 3.

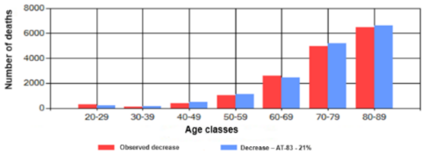

Fig. 3. Best GoF using QMD Test for all ages.

For this same entry, with a sample between 20 and 90 years and with the same age class division, using the QMD test, the table with the best GoF was the CSO 2001 FEMALE reduced by 6%. As it was not possible to find an adherent table by the calculation by age classes using the Qui-square test for the two entries proposed in the Exact Age calculation, with the sole objective of continuing with the research forcing adherence, it was decided experimentally to test other age intervals. From several tests, it was empirically obtained that the age interval between 22 and 87 years, corresponding to about 79% of the population data, allows a table to be adherent. Considering, therefore, as the lower class the sample included in the age range between 22 and 32 years old, four intermediate classes with a range of 11 years were established. The upper class was established between 72 and 87 years old. For this specific entry, the only adherent table through the Qui-square test was the GKM-70, reduced by 62%.

Subsequently, for comparison purposes, with the same interval entry between 22 and 87 years, and the same division of classes, the KS test was performed. Then, 2.559 adhered tables were obtained. The best GoF one was the BR - SEMsb-v.2010F, aggravated by 91%. The QMD test was also performed from the same interval between 22 and 87 years and the same division of classes. The table with the best GoF was the BR - SEMsb-v.2010F, aggravated by 81%. Table 4 summarizes the comparison of the number of tables adhered to in each test.

Table 4. Number of General Mortality Tables adhered to using each test.

Data \ Test	Qui-square	KS	QMD
The Entire Population	There was no GoF	1.203	12.667
Sample aged between 20 and 90 years	There was no GoF	1.936	12.667
Sample aged between 22 and 87 years	01	2.559	12.667

Table 5 highlights the tables that presented the best GoF in the calculation by age classes.

Table 5. General Mortality Tables with better GoF from each test.

Data \ Test	Qui-square	KS	QMD
The Entire Population	There was no GoF	CSO 2001FEMALE decreased by 6%	CSO 2001FEMALE decreased by 7%,
Sample aged between 20 and 90 years	There was no GoF	AT-83 decreased by 21%	CSO 2001FEMALE decreased by 6%
Sample aged between 22 and 87 years	GKM-70 decreased by 62%	BR - SEMsb-v.2010F increased by 91%	BR - SEMsb-v.2010F increased by 81%

4 Analysis of Results

The tests of the previous section demonstrated that the Qui-square test is not the most appropriate for this population sample. For any table to be adherent from this test, it was necessary to reduce the sampling space and, still, the number of adhered tables was minimal. On the other hand, the Kolmogorov-Smirnov and the Quadratic Measure Deviation tests proved to be applicable for the entire data sample.

Due to the different test statistics of each type of GoF test, it is difficult to compare the results of the best-adhered tables to each test applied differently. In the Qui-square test, the x^2 statistic is calculated and the best test result is the one with the lowest value of this statistic, among those whose value is lower than the critical value derived from the Qui-Square distribution table with a significance level of 5%. In the KS test, on other hand, the p-value statistic is calculated and the best test result is the one with the lowest value of this statistic, among those whose value is lower than the critical value derived from the KS distribution table with a significance level of 5%. In the Quadratic Measure Deviation test, the best result is the lowest QMD statistic. Therefore, these are algebraic calculations and distinct mathematical quantities.

During this research, it was not found in the literature or the social security market a specific way of performing such a comparison. Therefore, the authors chose to do a qualitative analysis of the data found. After the analysis, we verified that, for this database of Active, Inactive and Pensioners Mortality, the actuarial table with the best GoF was the AT-83-Male, decreased by 3 years, based on the Kolmogorov-Smirnov test.

5 Final Considerations

This research did not find in the literature or the social security market a recognized way of quantitatively comparing the results of actuarial tables in different GoF tests. This is because the different tests employ different test statistics, even in terms of mathematical quantities. Thus, the authors did a qualitative analysis of the data found, which was considered satisfactory for the objective of this specific study. It is worth mentioning, however, that the results presented in the research are sensitive to variations hypothesis, in the normative and cadastral basis.

This paper has shown that, although the Qui-square GoF test is already consecrated by its use in academic literature, it is not the one that best meets the specificities of the AF. It was concluded that, among the main GoF tests disseminated in the academia, the one that best applies for the selection of actuarial tables for the militaries of the Brazilian Armed Forces is the Kolmogorov-Smirnov test. This test proved to be robust, allowing the GoF of several actuarial tables, including a sample space of the population of the most comprehensive AF and with a satisfactory result of test statistics.

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