Using robust approach concept to solve the production planning problem in manufacturing systems

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Abstract. The use of Linear Programming (LP) models to plan production has been widely used to provide optimal solutions. However, LP models as well as models based on Material Requirements Planning (MRP) use deterministic parameters. In this context, one of the classic approaches to deal with a dynamic and uncertain scenario is the robust approach, which proposes a suboptimal solution through deterministic approaches capable of incorporating variations in the problem solutions. A parameter that is subject to such variation is the system's production capacity, since this parameter is directly impacted by the way the system is workloaded and its cycle times. Thus, to analyze the relationship among system's production capacity, cycle time and work-in-process, an alternative is modeling the production system as a queuing system using little's law. This research aims to solve the production planning problem modeled as a queuing system to propose managers a production planning model that uses efficient, simple and robust methods. Therefore, a theorem was proposed to prove the effectiveness of the used method. As a result, the mathematical model obtained allows the planner to make the use of a robust linear programming model, with low computational cost, capable of obtaining good quality solutions when compared to complex nonlinear programming problems. Finally, the model was submitted to a numerical experiment to better illustrate how it works with the inputs from an electronic component manufacturing company.

Keywords: Robust Optimization. Linear Programming. Convex Programming.

1 Introduction

Since 1960, sophisticated decision support systems have been implemented in a large number of medium and large organizations, based on Material Requirements Planning (MRP) systems. However, such systems only provide feasible solutions for the production planning problem, in the sense of not considering the accomplishment of optimality criterias, therefore the quality of the solution found does not provide an adequate analysis both regarding to costs and the problem optimality. In this scenario, mathematical programming models that use Linear Programming (LP) have been proposed to solve manufacturing problems in productive systems, along planning horizons. However, LP models consider that planning parameters are completely deterministic, thus excluding any possibility of parameter variation. Therefore, performing production planning efficiently is a big challenge, especially when, in the daily experience of the shop floor, there is, indeed, variability in the parameters that are used as input data in the models.

Given the parameters variability, another possibility is the use of stochastic models. However, the stochastic approach considerably increases the analytical and numerical complexity of the models, without necessarily ensuring better results. In this context, a classic approach to deal with a dynamic and uncertain scenario is the robust approach, which proposes a suboptimal solution through deterministic approaches capable of incorporating the action of variations in the solution of problems. Thus, to simulate the reality of the companies' shop floor through the production planning model, the concept of robustness will be incorporated into the mathematical model of production planning through the main parameters subject to such variations, such as times of queues, which are responsible for impacting production capacity estimates.

Production capacity is intrinsic and pre-set into each machine prescribed in the production system, known as nominal capacity. However, due to variations that the planning parameters are subject to, this nominal capacity is directly impacted. It consequently guides the manager to carry out a new capacity planning estimation considering such variations and setting and estimation of the real system capacity lower than the nominal capacity. Thus, it appears as an alternative, to model the production system from a queuing system perspective through Little's Law. Modeling production systems as a queuing problem leads to a relationship among the following variables, production rate (X), Work-in-process (WIP), and total cycle time (CT), i.e. processing time plus the total wait time in the system. When the production rate is the independent variable, the relation $X = WIP/CT$ is called Little's Law, and more generally in the production planning scope, it is called Clearing Function, which is a function used to estimate real capacity in production systems modeled as queuing problems. Most production planning models based on mathematical programming do not incorporate production capacity estimators as a decision variable. One of the main reasons for this is the non-linearity of the model, which makes it more difficult to use it daily on the shop floor. However, production capacity is directly impacted by the way the system is loaded and the cycle times spent on the system. Thus, these estimators, in general, appear in the model as an external parameter.

Therefore, this work seeks to solve the production planning problem modeled as a queuing system that incorporates capacity estimators as one of the decision variables to propose managers an efficient and robust planning model. The robust production planning model provided here allows the planner to make use of a simple and efficient production planning model. The proposed model combines robust optimization techniques with linear programming, to solve a convex optimization problem. Thus, allowing the feasibility of solving the problem in a certain range of variation for parameters under uncertainty.

2 Theoretical background

2.1 Robust optimization

Real-life optimization problems often contain uncertain data, e.g., demand variability, cycle times, setup times, productive capacity, etc. The reasons for these uncertainties in the data could come from measurement/estimation errors that come from lack of exact knowledge of the parameters of the mathematical model or because of the business' dynamics.

There are two distinct approaches for dealing with data uncertainty in optimization: robust and stochastic optimization. Stochastic optimization assumes an important premise, which is that the true probability distribution of uncertain data must be known or estimated. Robust optimization, on the other hand, does not assume that the probability distributions are known, but assumes that the uncertain data resides in a set of uncertainties [1].

Robust optimization is a relatively young research field and has been mainly developed in the last 15 years. Especially in the last 5 years, there have been many publications that show the value of robust optimization in applications in many fields including finance, management science, supply chain, healthcare and engineering ([2]; [3]; [4]; [5]; [6]; [7]; [8]; [9]; [10]).

[11] robust model was the pioneer in robust optimization and it is extremely conservative, in the sense that the value of the objective function deteriorates too much to guarantee the robustness, in terms of feasibility, of the solution. [11] used the term "uncertainty box" to refer to the space of realization of uncertain parameters in the model, that is, the vector space has as its center an average vector that can vary symmetrically over a given range (deviation) along the "box". The advantage of this approach is the simplicity of its application. The disadvantage is the high level of conservatism.

2.2 Clearing Function

The Clearing Function (CF) is a function used to estimate the real capacity of the production system and comes from an interpretation of Little's Law.

The Clearing Function was interpreted in different forms, and the figure below shows some of these possible approaches of interpretation and their respective authors.

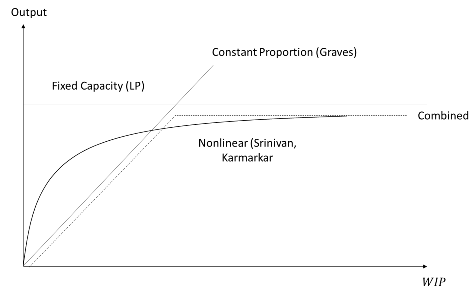

Fig. 1. Clearing Function Approaches.

The first approach is the "constant proportion" Clearing Function. This approach was proposed by [12] and allows unlimited outputs along a planning horizon, however, it considers a fixed Lead Time, similar to the presented case of MRP. The "Fixed capacity" Clearing Function or nominal capacity corresponds to a production that independent of level of WIP the production rate continues according to the system's capacity level. Furthermore, production is seen as instantaneous since it has no restriction with Lead Time. However, it has a maximum limit for the production rate over a planning horizon. Then, there is the "combined model" that uses both the "constant proportion" and the "Fixed capacity" approach, thus allowing for the advantages that both provide in the systems. Finally, the nonlinear Clearing Function model, which were introduced by [13] and [14]. This model considers the Clearing Function as a function dependent on the system's Work-in-process, that is, production rates are a non-linear function of the Work-in-process. Some research applied the different forms of the clearing function, for example, [15], [16], [17], [18], [19], [20], [21], [22], [23] and [24].

Recently, [25] explored other approaches to Little's Law, where she considers Clearing Function as a function of both Work-in-process and Lead Time. The assumption that the system's Work-in-process is the only argument of the Clearing Function is not enough to obtain a model capable of providing all the necessary information about the production rate and cycle times of the system. It is clear that production rates do not depend exclusively on the Work-in-process of the system, since several other times are involved in the system, such as: setup times, waiting times at workstations, time of processing in machines etc. The more extra time spent on these times, the lower the real available production capacity of the system. Thus, do not consider them when modeling a production system can compromise the model's effectiveness.

In general, theoretical models of production planning predict that the production rate depends on the state of the system's workload. It can be confirmed with practical perceptions of the shop floor daily routine. However, the work-in-process of the system also affects the production cycle time and the production rate of a system, and, therefore, it can be said that these variables are clearly related. Thus, it is a fact that Little's law needs to be explored considering this reality. Little's law is used to show the relationship among the following variables, WIP (Average System Work-in-process), Average System Cycle Time (CT) and Average System Production Rate (X) [26]. It is possible to see that the same level of production rate can be achieved by different levels of WIP and cycle time. This implies that the level of production rate in a productive system is completely undetermined, since there are two variables

$$X = \frac{WIP}{CT} = \frac{\alpha \times WIP}{\alpha \times CT}, \forall \alpha \neq 0 \tag{1}$$

Therefore, Little's law is a point-set relation, generating a cloud of points on the graph, that is, for the same WIP level there are several different production rates, showing that, in fact, instead of little's law being a function, it is a point-set relationship.

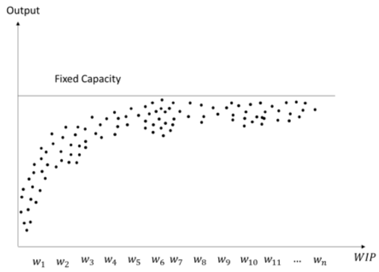

Fig. 2. Little's law as a point-set relationship for each WIP.

It is observed that the Average Cycle Time and the average Work-in-process of the system are directly related variables, that is, the higher the level of Work-in-process, the greater will be the average cycle time to complete the task, and vice versa, as cycle time is a function of Work-in-process. However, cycle time is not a linear Work-in-process function. Considering that, an adaptation of Little's Law was proposed by [25].

$$X\big(WIP, CT(WIP)\big) = \frac{WIP}{CT(WIP)} \tag{2}$$

[27] presented that the Clearing Function approach, as a two variables dependent function (average work-in-process and average cycle time in the system) is an efficient function to estimate system's production capacity, using dynamic regression to estimate the Clearing Function. [27] demonstrated through a planning of experiments and a simulation in a food factory that after the Clearing Function adjustments via dynamic regression, both for the functional case $X(WIP)$ (Clearing Function as a function of a variable), as the new proposed model $X(WIP, S)$ (Clearing Function as a function of two variables), proved to be efficient independent of the number of variables involved in the process.

A content analysis was performed on the Scopus database, no year restriction, with the terms "Production Planning Problem", "Mathematical Model" and "Robust". Only 9 scientific articles were obtained that deal with production planning via robust optimization. 4 of them analyzed "demand" as an uncertain parameter, or robust parameter, 3 of them analyzed the "production level", 1 analyzed the "budget available for production" and 1 the "assembly time". With that, we verified a gap in the literature regarding robust optimization analysis in parameters associated with production capacity.

3 Approximation of robust convex mathematical programming models by linear programming models

3.1 Approximation Theory

The hypotheses of the approximation theory of this research are as follows ([28]; [29]):

Let $\varphi: \mathbb{R}^n \to \mathbb{R}$ a concave function, $\varphi \in C^1(\mathbb{R}^n)$ and let a convex set Ω_1 formed by the intersection of the hypograph of the φ function and the set $x \geq 0$.

The illustration in $\mathbb{R}^2$ of the interval $I = [a, b]$ can be seen in the following figure.

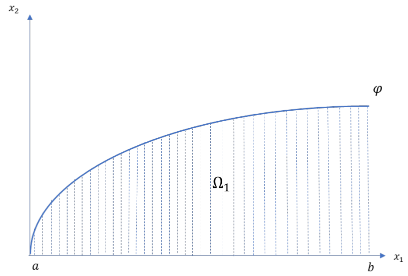

Fig. 3. Illustration set Ω_1.

Let $\varphi: \mathbb{R}^n \to \mathbb{R}$ a concave function, $\varphi \in C^1(\mathbb{R}^n)$ and let $x_1, x_2 \in \mathbb{R}^n$; Let $x = \lambda x_1 + (1-\lambda)x_2, \lambda \in [0,1]$, the line segment connecting points x_1 e x_2; Let $g_i: \mathbb{R}^n \to \mathbb{R}, i = 1, \dots m-1$, a set of affine functions. Let the following partition of the range $I = [a, b]$:

$$a = a_i < a_{i+1} < a_{i+2} < a_{i+3} < \cdots < a_n = b \tag{3}$$

Let $g(x)$ defined by:

$$g(x) = \sum_{i=0}^{m-1} g_i(x) \tag{4}$$

Where $g_i(x)$ is given by:

$$g_i(x) = \{ \begin{matrix} m_i x + c_i, & a \leq x \leq b \\ 0, & otherwise \end{matrix} \tag{5}$$

Where m_i and c_i represent the parameters related to the angular and linear coefficient, respectively, of each line segment.

Let, $\Omega_2 = \{(x, \mu): \mu \leq g(x), x \in \mathbb{R}^n, x \in [a, b], a = a_i < a_{i+1} < a_{i+2} < \cdots < a_n = b\}$.

The illustration on $\mathbb{R}^2$ can be seen in the following figure,

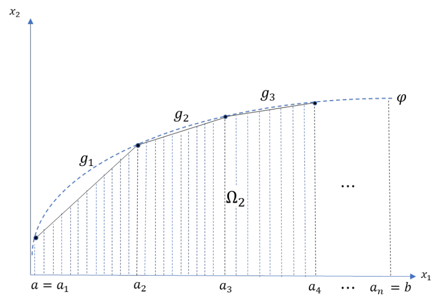

Fig. 4. Illustration set Ω_2.

Then, we have the concave function φ approximated by a set of $g_i(x)$ concave.

In order to support this statement, two lemmas are presented below.

Lemma 1: Let the convex set Ω_1 as defined in figure 3, and the convex polyhedral set Ω_2 as defined in figure 4 and let $\Omega_2 \subset \Omega_1$. Let $a = a_1 < a_2 < a_3 < \cdots < a_n = b$ be a partition of the closed interval $[a, b]$ as shown in figure 4. Then, when n tends to infinity, the convex polyhedral set Ω_2 tends to the convex set Ω_1.

Proof: The proof of lemma 1 comes from the concepts of the Fundamental Theorem of Calculus.

Lemma 2: Let the convex set Ω_2 be defined in figure 4, and $n_0, n \in \mathbb{N}$, where $n_0 \leq n$. Let $a = x_1 < x_2 < \cdots < x_n = b$ be the partition of the closed interval $[a, b]$ where the points are defined by the convex combination between the extreme points. Then,

$$\begin{array}{c} \max g(x) \\ x \in \Omega_2 \\ n_0 < n \end{array} \quad \leq \quad \begin{array}{c} \max g(x) \\ x \in \Omega_2 \\ n_0 = n \end{array} \tag{6}$$

Proof: The proof of Lemma 2 can be obtained in [30].

Theorem: Let it be a convex (P) programming problem, given by:

$$\begin{array}{ll} & \max\ Z(x, W) = c^T x + d^T W \\ \textit{Subject to}: & Ax \leq b \\ & x \leq \varphi(W) \\ & x \geq 0 \end{array} \tag{P}$$

Where Z: $\mathbb{R}^n \to \mathbb{R}$, and $c \in \mathbb{R}^n$, $d \in \mathbb{R}^n$, $x \in \mathbb{R}^n, W \in \mathbb{R}^n$, $A \in \mathbb{R}^n \to \mathbb{R}^m$, $b \in \mathbb{R}^m$, $\varphi(W)$: $\mathbb{R}^n \to \mathbb{R}$, concave and $\varphi(W) \in C^1(\mathbb{R}^n)$.

Let a set of linear programming problems (L), given by:

$$\begin{array}{lll} & \max\ Z(x, W) = c^T x + d^T W & \\ \textit{Subject to}: & Ax \leq b & \\ & x_i \leq g_i(W) & \forall_i = 1, \ldots, n. \\ & x \geq 0 & \end{array} \tag{L}$$

Where Z: $\mathbb{R}^n \to \mathbb{R}$, $c \in \mathbb{R}^n$, $d \in \mathbb{R}^n$, $x \in \mathbb{R}^n$, $W \in \mathbb{R}^n$, $A \in \mathbb{R}^n \to \mathbb{R}^m$, $b \in \mathbb{R}^m$ and g_i: $\mathbb{R}^n \to \mathbb{R}$, linear, $i = 1, \ldots, m - 1$.

Then, we have that the result of the convex programming problem (P) can be arbitrarily approximated by the result of the linear programming problem (L).

3.2 Approximation theory applied to production planning models

Mathematical Production Planning Model

The description of the planning model presented in this research considers the schemes proposed by [31], by [32] and [33]. The description of the production planning problem is as follows: The indexes are for products $i = 1, 2, \ldots, N$; for the components $k = 1, 2, \ldots, K$ and for the discrete periods of the planning horizon $j = 1, 2, \ldots, T$. The following parameters are: b_{ki} the number of components k used to produce one unit of product i; h_i the standard time required to produce one unit of product i; Z_j the labor (in standard time units) available to be consumed in period j. S_{kj} the quantity of supplies of component k available to be consumed in period j; D_{ij} the minimum demand for product i in period j. γ_{ij} the nominal capacity of product i in period j; c_{ij}, h_{ij}, w_{ij} and r_{ij} the cost of production, inventory, work-in-process and releases of each unit of product i in period j, respectively. Decision Variables x_{ij}, I_{ij}, W_{ij} and R_{ij} respectively represent the average level of production, inventory, work-in-process and releases of product i in period j. The mathematical model of production planning (7) is as follows:

$$Min \ \sum_{j=1}^{T} \sum_{i=1}^{N} c_{ij}x_{ij} + h_{ij}I_{ij} + w_{ij}W_{ij} + r_{ij}R_{ij} \tag{7.1}$$

$$s.t. \quad \sum_{j=1}^{t} \sum_{i=1}^{N} b_{ki}x_{ij} \le \sum_{j=1}^{t} S_{kj} \qquad k = 1,2,\dots,K, \ t = 1,2,\dots,T \tag{7.2}$$

$$I_{ij} = I_{ij-1} + x_{ij} - D_{ij} \qquad i = 1,2,\dots,N, \ j = 1,2,\dots,T \tag{7.3}$$

$$W_{ij} = W_{ij-1} + R_{ij} - x_{ij} \qquad i = 1,2,\dots,N, \ j = 1,2,\dots,T \tag{7.4}$$

$$x_{ij} \ge D_{ij} \qquad i = 1,2,\dots,N \ \ j = 1,2,\dots,T \tag{7.5}$$

$$\sum_{i=1}^{N} h_i x_{ij} \le Z_j \qquad j = 1,2,\dots,T \tag{7.6}$$

$$x_{ij} - \gamma_{ij} \le 0 \qquad i = 1,2,\dots,N \ \ j = 1,2,\dots,T \tag{7.7}$$

$$x_{ij} \ge 0, I_{ij} \ge 0, W_{ij} \ge 0, R_{ij} \ge 0 \qquad i = 1,2,\dots,N, \ j = 1,2,\dots,T \tag{7.8}$$

The objective function (7.1) of the model seeks to obtain the lowest possible cost from the optimal combination of the variables x_{ij}, I_{ij}, W_{ij} and R_{ij}. The Bill of Material – BOM (7.2) restriction seeks to ensure that the quantity of components used in the production of each product is less than or equal to the total material resources available in period j. Constraints 7.3 and 7.4 refer to the balance of inventory and work-in-process, respectively, of product i in period j. Constraint 7.5 refers to the demand to be met for each product i at the end of period j. Constraint 7.6 refers to the availability of labor resources (in standard time units) in each period j to be consumed for the production of products i. Constraint 7.7 refers to the capacity to be considered by the production of products i in each period j. Constraint 7.8 refers to the problem's non-negativity constraints.

Mathematical model using queuing system concepts

In the previous model of mathematical programming (7) the concepts of Little's Law will be incorporated, that is, it seeks to modify the model's capacity definition parameter, which was previously fixed, by a concave function, called Clearing Function.

The purpose of this introduction in the model is due to the fact that the vast majority of production planning models based on mathematical programming incorporate production capacity as a parameter and do not consider that as a variable. Therefore, the production planning model of this work considered the use of the Clearing Function, ($\varphi_{ij}(W_{ij})$), to estimate the system capacity, replacing constraint 07 of the model by:

$$x_{ij} - \varphi_{ij}(W_{ij}) \leq 0 \qquad (8)$$

The production planning model considers the Clearing Function as proposed by [14]. Where $\varphi_{ij}(W_{ij}) = \frac{n \times N_{ij} \times W_{ij}}{N_{ij} \times S_{ij} + W_{ij}}, \forall i,\ \forall j$, and n represents the number of workstations, and N is the nominal capacity of the machine. With the modification, the linearity of the model is not preserved, since the Clearing Function is a concave function, so there is a convex programming model.

Approximation of Clearing Function using affine Functions

Due to the complexity that has come from a non-linear programming problem, this section seeks to present how to approximate the concave function, Clearing Function, using a set of affine functions. Let the Clearing Function as a function of the average Work-in-process of the system be represented by $\varphi(WIP)$: $\mathbb{R}^n \to \mathbb{R}$ a concave function, $\varphi(WIP) \in C^1(\mathbb{R}^n)$. Let $g_i(WIP)$: $\mathbb{R}^n \to \mathbb{R}, i = 1, \dots m-1$, be a set of affine functions, then the Clearing Function will be approximated through a set of affine functions as follows:

$$\varphi(WIP) = \min_i \{g_i(WIP)\}\ i = 1, \dots m-1 \qquad (9)$$

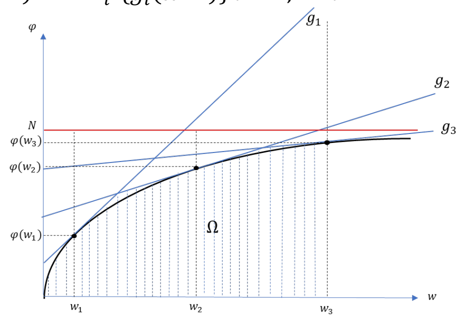

Fig. 5. Illustration of the Clearing Function approach to affine functions.

In the figure 5, there is an example for three average levels of Work-in-process (w_1, w_2 e w_3), where the curve in black bold represents the concave function, Clearing Function, as a function of the Work-in-process levels system mean ($\varphi(WIP)$) and the segments in blue (g_1, g_2 e g_3) indicate the affine functions used to approximate of the concave function.

Thus, it is noted that the Clearing Function is defined by the affine function that presents the minimum value among the affine functions in each level of Work-in-process. In addition to the 3 segments, the figure 5 shows a segment with equal zero slope (N), in red, representing the nominal capacity of the system, which aims to ensure that the system output is obtained by the Clearing Function and do not exceed its nominal capacity, which needs also to be considered. It gives the following definition of the approximate Clearing Function at each work-in-process level:

$$\varphi(w_1) = \min \{g_1(w_1), g_2(w_1), g_3(w_1), N\} \quad (10)$$
$$\varphi(w_2) = \min \{g_1(w_2), g_2(w_2), g_3(w_2), N\}$$
$$\varphi(w_3) = \min \{g_1(w_3), g_2(w_3), g_3(w_3), N\}$$

Mathematical model considering the approximation of Clearing Function through affine functions

By making the use of the Clearing Function concept using affine functions, it is possible to make the convex programming model be solved as a linear programming model.

Therefore, the production planning model of this work considered the use of affine functions ($g_{ij}(W_{ij})$) to estimate the system capacity by replacing again the constraint 07 of the model by:

$$x_{ij} - g_{ij}(W_{ij}) \leq 0 \quad (11)$$

With this modification, the linearity of the model is preserved.

3.3 Application

The purpose of this application section is to explore a numerical experiment using the proposed approximation theory with the inputs obtained from an electronic component manufacturing company. This experiment seeks to optimize production planning considering the production of 3 batches of products, with two types of components required to produce each, over a 12-period planning horizon. Waiting times in queue were considered stochastic. Applying the robust convex programming model, the objective function value of 22,031.00 monetary units was obtained. When compared to the convex programming model without the presence of robustness, there was an increase of 6.9%. Applying the robust linear programming model, the objective function value was 22,437.00 monetary units. The difference between the two robust models was only 1.84%. What can be confirmed the effectiveness of the robust convex programming model approach using a set of affine functions. Despite the increase in total costs, the model presented a suboptimal solution that allows the model to be feasible in every region of uncertainty of the parameter that was inserted the robustness.

4 Final Considerations

Therefore, it is possible to solve complex non-linear programming problems using simpler linear programming (LP) models with the equivalent effectiveness. Making use of a simple method such as LP allows greater acceptance of implementation on the shop floor, due to the lack of specialized labor.

However, such systems only provide feasible solutions for the production planning problem, in the sense of not considering the accomplishment of optimality criterias, therefore the quality of the solution found does not provide an adequate analysis both regarding to costs and the problem optimality.

In the case addressed in this work, for simplification reasons, it was admitted that only the waiting times in queue are stochastic, but the approach does not change even when considering that all the parameters of the model may vary.

Therefore, the use of robust scheduling to solve production planning problems is a powerful tool to support decision making in view of the variability of planning parameters.

Finally, the computational cost of this approach is not much higher than the computational cost of the deterministic approach, which makes this approach attractive to use.

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