Optimization of the Logistics Chain in the Export of Food Products using concepts of Transportation Problems

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Abstract. Companies face significant challenges in managing their distribution networks, especially due to the complexity and constant variations in planning parameters, such as production quantity and fluctuating product sales. However, mathematical models, such as minimum-cost flow models based on operations research, can help address this complexity and uncertainty. In this study, we proposed a minimum-cost flow model to assist companies in making optimal decisions regarding the distribution routes and quantities of finished products. The model was developed using LINGO software and was highly effective in identifying the best routes and product quantities for transportation while minimizing total costs. Through the integration of this model into companies' distribution planning, significant efficiency and cost savings can be achieved. This study highlights the potential of mathematical models in revolutionizing distribution networks and their value in the decision-making process.

Keywords: Logistics, Operational Research, Transports, Cost Minimization.

1 Introduction

In today's competitive and globalized world, companies face intense competition and increasingly demanding consumers. Therefore, the ability to make assertive decisions is crucial. Efficient management can provide a significant competitive advantage, given the complexity of the processes involved. For companies engaged in goods distribution, logistics, especially transportation within the distribution network, plays an indispensable role in achieving business success. By reducing costs, efficient management offers more accessible services, resulting in lower final prices, faster deliveries, and higher customer satisfaction.

A distribution network encompasses various stages, from source origins to final destinations, through distribution centers and intermediate locations. The challenge lies in finding the best route, choosing the appropriate transportation modes, efficiently meeting demands, and minimizing costs and time. To achieve this, decision-making can be based on managerial intuition or mathematical modeling processes, which allow simulating different scenarios and finding optimal solutions.

Operations research (OR) is a scientific approach that structures processes and proposes alternatives and solutions through mathematical models, enabling analysis and comparison of the considered parameters, as time and total cost.

The objective of this study is to propose a mathematical transportation model to optimize the logistics network of the export process of a food company, aiming to reduce distribution time, resources used, and costs involved. To achieve this goal, the following steps were outlined: mapping the company's export logistics process, formulating and applying the mathematical model, and analyzing the results obtained.

2 Methodological Procedures

A methodological process begins with the literature review, which includes searching for works in research bases that have as keywords: "export", "operational research" and "capacitated minimum-cost flow problems". This phase provides a theoretical foundation for the study and ensures a comprehensive understanding of the context. To guide the research efforts, the Scopus database was selected. The search for "capacitated minimum-cost flow problems" yielded 6 publications across the fields of Computer Science, Decision Making, Mathematics, Engineering, and Business, Management and Accounting.

Analysis of the retrieved publications revealed a predominant focus on proposing algorithms for solving flow network problems, albeit with limited real-world applications. Consequently, this study distinguishes itself by adopting a holistic approach that not only addresses the theoretical complexities associated with exportation issues but also emphasizes the practical application of Operations Research methodologies in supporting decision-making processes within companies. To underscore the significance of quantitative analysis and the pursuit of optimal solutions, a real-world case study is conducted. By examining a tangible exportation scenario, this study aims to demonstrate the relevance and efficacy of Operations Research techniques in addressing complex logistical challenges and optimizing decision outcomes.

The next step is to analyze the exporting process of the food company, divided into three distinct phases:

During the initial phase of the process, termed the study phase, it is important to first gain a thorough understanding of the actual stage of the export process. This includes identifying the main bottlenecks, defining the study parameters, understanding how the various modes of transportation are used and analyzing how different actors within the process interact with each other. In parallel, comprehend how to develop and apply a minimum-cost flow problem based on the specificities of the organization.

During the diagnostic phase, a quantitative analysis of the company's data is conducted. This analysis includes collecting detailed data from the company regarding the process of exporting goods from their factories to external market customers, which includes the location of each factory, the quantity of products each factory can produce, the international market customers, and their respective demands, etc. To develop a mathematical model that accurately represents the logistics flow, operations research methods are applied. This involves meticulous data collection and analysis.

Lastly, in the propositional phase, a mathematical model is developed based on data analysis and manipulation to accurately represent the logistic flow. With the operational research software LINGO, critical analysis of the results obtained is performed, to support the results obtained. In this stage, the proposed solution is verified and validated, aiming to identify possible improvements and necessary adjustments to optimize the export process of the food company.

3 Literature Review

3.1 Supply Chain Management

Supply Chain Management (SCM) is a crucial component of contemporary business operations, encompassing the planning, coordination, and control of the flow of goods, services, and information from initial raw material sourcing to final customer delivery [1]. In today's fiercely competitive and globalized marketplace, efficient and effective SCM plays a pivotal role in enhancing an organization's competitiveness, customer satisfaction, and overall profitability. By optimizing the entire supply chain, companies can streamline processes, reduce lead times, minimize inventory levels, and improve overall responsiveness to customer demands [2] [3].

Effective SCM involves the integration of various elements, including suppliers, manufacturers, distributors, retailers, and customers. This integration requires seamless information sharing and collaboration among all stakeholders, which can be facilitated through advanced technologies and communication systems. Key components of SCM include demand forecasting, inventory management, production planning, logistics, and distribution [4]. Implementing the right SCM strategies and methodologies enables companies to achieve cost savings, improve service levels, and gain a competitive advantage in the market [5].

3.2 Operations Research

Many authors define Operations Research (OR) as a method for decision-making. According to [6], OR consists of describing an organized system that, through a mathematical model and experimentation, finds the best way to operate the system. [7] classify operations research as a science of knowledge that structures processes and proposes a set of alternatives and actions. According to [8], operations research is a tool for solving real problems; making decisions based on quantitative data and correlations; planning and operating systems using technology and methods from other spheres of knowledge; and finding the optimal solution to the problem, reducing costs, and increasing profit.

3.3 Minimum Cost Flow Problems

The minimum cost flow problem is a significant optimization problem for distribution networks. According to [9], includes specific concerns like transportation, assignment, shortest path, and maximum flow. A practical application of this problem is planning a distribution operation with transshipment points, which involves determining an optimal route plan to transport goods from origins to destinations, passing through transshipment points.

The central objective of the minimum cost flow problem – also considered as a transshipment transportation problem – is to minimize the total cost of distributing inputs from the origin to the destination through intermediate points. The constraints are established to ensure that each origin dispatches at most all its supply and that all demands of the destinations are met. For intermediate points, the constraints impose that the quantity received is equal to the quantity sent, acting only as a passing node and ensuring flow conservation. The last set of constraints prevents the model from yielding negative quantities to be transported as a result, thus all quantities transported must be non-negative [9].

According to [10], the way to solve network problems with inequalities is by creating artificial variables. When dealing with a problem where the supply is greater than the demand, meaning there are excess goods in stock, the solution is to create artificial demand units, only for calculation purposes, to equalize the total quantities of supply and demand. Conversely, if there is insufficient production capacity to meet all the necessary demand, additional supply points are created so that the excess demand can be met. The variables created are only to facilitate calculations in the matrix, and these variables do not affect the final cost in the objective function.

4 Characteristics of the Problem

4.1 The Company

The food industry under scrutiny operates 18 factories spread across Brazil, producing goods for both domestic and international markets. This study focuses on the international distribution of three refrigerated finished products: products *X, Y*, and *Z*. The primary overseas markets are segmented according to specific maritime routes, covering regions such as Africa, the Americas, Europe, the Far East, the South Pacific, Japan, and the Middle East.

The complexity of the logistics chain is evident, with the company controlling operations from farms to animal feed manufacturing, resulting in production that fluctuates monthly due to the availability of animals for slaughter. Factory production capacity fluctuates accordingly, as does demand from overseas markets.

Due to limited space in the factories to store unsold products, the company utilizes two third-party warehouses near ports to store export-bound products. Road transport, using refrigerated trucks, is employed to move products from the factories to the warehouses, while container trucks are used to transport products to the ports, where they are loaded onto container ships for delivery to overseas customers.

The primary logistical challenge is to plan monthly distribution of finished products between factories and third-party warehouses, considering capacity constraints and costs. To assist in this decision-making process, a mathematical model will be proposed to analyze different distribution scenarios, aiming to minimize total costs while considering all relevant variables and constraints.

4.2 Data Collection

Data acquisition was obtained through meetings with the freight procurement team, exportation team, and warehouse team, as well as the production process planning and sales team for the company's international market. To ensure data confidentiality, the data used in this work reflects the real parameters of the company; however, a random deflator index was applied to alter the values while maintaining proportionality and not compromising the accuracy and efficiency of the model.

The input parameters of the model are 18 factories, which are origin nodes, offering your respective number of products; 2 warehouses that can act as both supply and demand nodes, depending on the month, whether there is a need to deposit or withdraw products; 4 ports, which are transshipment nodes; and 7 customers in the international market (IM), demand nodes that only receive goods. In addition to the nodes in the distribution chain for the international market, the model parameters also include the capacities of each node, the 3 products, the 3 modes of transportation, and the costs for each possible route.

The locations of each node in the distribution chain are presented in Table 1, containing the city and state of each factory, warehouse, port, and customer in the international market.

Table 1. Locations of each node in the supply chain.

Node	Description	Location
1	*Factory 1*	*State of Goiás - GO*
2	*Factory 2*	*State of Goiás - GO*
3	*Factory 3*	*State of Goiás - GO*
4	*Factory 4*	*State of Goiás - GO*
5	*Factory 5*	*State of Minas Gerais - MG*
6	*Factory 6*	*State of Mato Grosso do Sul - MS*
7	*Factory 7*	*State of Mato Grosso - MT*
8	*Factory 8*	*State of Mato Grosso - MT*
9	*Factory 9*	*State of Paraná - PR*
10	*Factory 10*	*State of Paraná - PR*
11	*Factory 11*	*State of Paraná - PR*
12	*Factory 12*	*State of Rio Grande do Sul - RS*
13	*Factory 13*	*State of Rio Grande do Sul - RS*
14	*Factory 14*	*State of Rio Grande do Sul - RS*
15	*Factory 15*	*State of Santa Catarina - SC*
16	*Factory 16*	*State of Santa Catarina - SC*
17	*Factory 17*	*State of Santa Catarina - SC*
18	*Factory 18*	*State of Santa Catarina - SC*
19	*Warehouse 1*	*City of Paranaguá - PR*
20	*Warehouse 2*	*City of Itajaí - SC*
21	*Port 1*	*City of Itajaí - SC*
22	*Port 2*	*City of Navegantes - SC*
23	*Port 3*	*City of Paranaguá - PR*
24	*Port 4*	*City of Rio Grande - RS*
25	*Foreign Market Customer 1*	*Africa*
26	*Foreign Market Customer 2*	*Americas*
27	*Foreign Market Customer 3*	*Europe*
28	*Foreign Market Customer 4*	*Far East*
29	*Foreign Market Customer 5*	*South Pacific*
30	*Foreign Market Customer 6*	*Japan*
31	*Foreign Market Customer 7*	*Middle East*

4.2.1 CAPACITY AND DEMAND

Data on production and demand were gathered for a period of one year from the company, and an average was calculated to use as a basis in solving the mathematical model, emphasizing that these values were distorted for information security.

Table 2 presents the data of the average production capacities of each factory (in tons) for each of the three products (*X*, *Y*, and *Z*), indicating which node in the chain each factory represents and the state in which they are located.

Table 2. Production capacity of each factory, in tons.

Node	State	Product X	Product Y	Product Z
1	*GO*	*10,030*	*0*	*0*
2	*GO*	*25,624*	*884*	*4,639*
3	*GO*	*0*	*913*	*0*
4	*GO*	*19,852*	*0*	*4,668*
5	*MG*	*4,821*	*649*	*2,076*
6	*MS*	*12,369*	*0*	*0*

7	*MT*	*39,594*	*0*	*3,042*
8	*MT*	*0*	*0*	*1,454*
9	*PR*	*16,834*	*0*	*0*
10	*PR*	*63,694*	*7,306*	*0*
11	*PR*	*13,661*	*0*	*4,497*
12	*RS*	*27,968*	*0*	*4,764*
13	*RS*	*15,110*	*0*	*0*
14	*RS*	*12,605*	*0*	*0*
15	*SC*	*0*	*0*	*11,201*
16	*SC*	*22,249*	*0*	*11,201*
17	*SC*	*12,771*	*4,655*	*0*
18	*SC*	*12,771*	*0*	*2,272*
Total Production		**309,953**	**14,407**	**49,814**

Table 3 presents the average quantity demanded by each customer in the international market (in tons) for each of the three products (*X*, *Y,* and *Z*), indicating which node in the chain each customer represents and their location.

Table 3. Demand of international market customers, in tons.

Node	International Market Customers	Product X	Product Y	Product Z
25	*Africa*	*14,002*	*5,880*	*4,912*
26	*Americas*	*10,786*	*4,366*	*482*
27	*Europe*	*13,658*	*1,461*	*170*
28	*Far East*	*21,970*	*147*	*23,525*
29	*South Pacific*	*9,516*	*1,916*	*2,416*
30	*Japan*	*27,056*	*0*	*12*
31	*Middle East*	*183,083*	*637*	*0*
	Total Demand	**280.071**	**14.407**	**31.517**

Table 4 displays the maximum export capacity of each port in tons, indicating which node in the chain each port represents. This maximum quantity (data that makes the problem capacitated) was determined through negotiations between the company and the shipping companies and may vary with each new negotiation.

Table 4. Maximum export capacity of each port terminal, in tons

Node	Port	State	Maximum Capacity
21	*Itajaí*	*SC*	*250,000*
22	*Navegantes*	*SC*	*45,000*
23	*Paranaguá*	*PR*	*40,000*
24	*Rio Grande*	*RS*	*50,000*

Table 5 compares the average total production quantity of all factories with the total demand quantity of all customers, by product and in tons. This comparison is performed to determine whether the third-party warehouses will be used as origins or destinations for products.

Table 5. The average quantity of each product stored, in tons.

	Product X	Product Y	Product Z
Total Production	*309,953*	*14,407*	*49,814*
Total Demand	*280,071*	*14,407*	*31,517*

Stored Quantity	29,882	0	18,297

In this example with the data being used, the warehouses will be considered destination nodes. As can be observed, the total quantity of supply for the two products exceeded the demand quantity, requiring the unsold quantity to be stored in one of the third-party warehouses.

Therefore, nodes 19 and 20, representing the warehouses in Paranaguá and Itajaí respectively, must together store 29,882 tons of product *X* and 18,287 tons of product *Y*. The problem solution will determine how much each warehouse will separately receive each of the three products.

4.2.2 LOGISTIC COSTS

Table 6 presents the prices for the routes from the factories (origin nodes) with their locations in the state to the two warehouses (destination nodes) - Paranaguá and Itajaí. The transportation between the factories to the warehouse is made by a certain type of refrigerated truck (mode 2), it considers the average price of freight for road transport by this type of refrigerated truck, plus costs associated with inbound movement services and storage of goods in the respective warehouses.

Table 6. Average price of routes from factories to warehouses, per ton (mode 2).

Origin Nodes		Destination Nodes	
Node	Factory	19 Node (Paranaguá)	20 Node (Itajaí)
1	*GO*	*R$ 18.36*	*R$ 16.51*
2	*GO*	*R$ 19.62*	*R$ 17.96*
3	*GO*	*R$ 21.16*	*R$ 19.05*
4	*GO*	*R$ 20.22*	*R$ 17.69*
5	*MG*	*R$ 16.50*	*R$ 14.62*
6	*MS*	*R$ 19.93*	*R$ 17.05*
7	*MT*	*R$ 47.09*	*R$ 44.69*
8	*MT*	*R$ 36.99*	*R$ 35.84*
9	*PR*	*R$ 5.07*	*R$ 7.15*
10	*PR*	*R$ 12.37*	*R$ 12.77*
11	*PR*	*R$ 11.81*	*R$ 13.68*
12	*RS*	*R$ 15.88*	*R$ 11.78*
13	*RS*	*R$ 12.46*	*R$ 10.63*
14	*RS*	*R$ 13.15*	*R$ 11.69*
15	*SC*	*R$ 8.65*	*R$ 6.91*
16	*SC*	*R$ 9.32*	*R$ 8.18*
17	*SC*	*R$ 10.22*	*R$ 9.63*
18	*SC*	*R$ 8.56*	*R$ 8.57*

Table 7 displays the prices for routes from the factories (origin nodes) to the four ports (destination nodes). This transportation is made by a different refrigerated truck (mode 1), considering the average price of freight for road, plus the previously mentioned terminal fees at the ports. Other ports in Brazil are not being considered in the model of this work due to external negotiation reasons, they do not provide services to the studied company.

Table 7. Average price of routes from factories to warehouses, per ton (mode 1).

Origin Nodes	Destination Nodes

Node	Factory	21 Node (Itajaí)	22 Node (Navegantes)	23 Node (Paranaguá)	24 Node (Rio Grande)
1	*GO*	*R$ 34.61*	*R$ 34.31*	*R$ 34.10*	*R$ 35.10*
2	*GO*	*R$ 36.19*	*R$ 36.51*	*R$ 34.48*	*R$ 37.00*
3	*GO*	*R$ 37.64*	*R$ 37.97*	*R$ 35.57*	*R$ 38.20*
4	*GO*	*R$ 35.58*	*R$ 34.33*	*R$ 34.26*	*R$ 37.80*
5	*MG*	*R$ 30.88*	*R$ 30.32*	*R$ 29.69*	*R$ 32.00*
6	*MS*	*R$ 30.74*	*R$ 29.84*	*R$ 29.56*	*R$ 30.49*
7	*MT*	*R$ 58.49*	*R$ 57.98*	*R$ 55.93*	*R$ 40.00*
8	*MT*	*R$ 48.00*	*R$ 49.30*	*R$ 47.44*	*R$ 51.00*
9	*PR*	*R$ 11.73*	*R$ 11.69*	*R$ 10.78*	*R$ 13.45*
10	*PR*	*R$ 15.26*	*R$ 15.95*	*R$ 15.78*	*R$ 21.84*
11	*PR*	*R$ 23.05*	*R$ 22.81*	*R$ 22.79*	*R$ 23.90*
12	*RS*	*R$ 14.37*	*R$ 14.21*	*R$ 17.38*	*R$ 12.73*
13	*RS*	*R$ 13.27*	*R$ 13.87*	*R$ 15.57*	*R$ 14.79*
14	*RS*	*R$ 14.61*	*R$ 14.74*	*R$ 16.24*	*R$ 14.37*
15	*SC*	*R$ 10.27*	*R$ 10.21*	*R$ 13.27*	*R$ 12.50*
16	*SC*	*R$ 12.15*	*R$ 11.63*	*R$ 13.97*	*R$ 14.00*
17	*SC*	*R$ 15.31*	*R$ 15.31*	*R$ 16.35*	*R$ 19.24*
18	*SC*	*R$ 12.12*	*R$ 12.02*	*R$ 13.39*	*R$ 18.24*

From factories to warehouses, refrigerated trucks are used as the mode of transportation (mode 2), as they can carry larger quantities (an average of 28 tons per truck) and do not incur container rental fees. For routes from factories to ports, goods are transported by container trucks (mode 1) to avoid multiple loading and unloading processes. Loading goods onto refrigerated trucks and stuffing containers are services performed by the company's employees at the factories, incurring no additional cost to the route.

Table 8 presents the prices for routes from Brazilian ports (origin nodes) to the seven international market customers (destination nodes), considering the average maritime freight rates by different shipping companies. These routes are carried out via container ships (mode 3). Unloading at the destination and distribution from foreign ports to consumer countries or cities are beyond the scope of this work and the responsibility of buyers. Additionally, documentation fees and customs duties are not considered.

Table 8. Average freight price from port terminals to international market customers, per ton (mode 3).

Origin Node		Destination Node						
Node	**Port**	**Africa**	**Americas**	**Europe**	**Far East**	**South Pacific**	**Japan**	**Middle East**
21	*Itajaí*	*R$ 20.80*	*R$ 15.68*	*R$ 18.05*	*R$ 28.16*	*R$ 16.57*	*R$ 31.01*	*R$ 22.06*
22	*Navegantes*	*R$ 22.80*	*R$ 20.40*	*R$ 17.95*	*R$ 27.73*	*R$ 19.06*	*R$ 30.94*	*R$ 23.67*
23	*Paranaguá*	*R$ 19.85*	*R$ 18.19*	*R$ 16.72*	*R$ 27.97*	*R$ 19.93*	*R$ 32.14*	*R$ 23.56*
24	*Rio Grande*	*R$ 23.83*	*R$ 24.10*	*R$ 24.48*	*R$ 26.52*	*R$ 26.52*	*R$ 33.10*	*R$ 24.60*

5. MATHEMATICAL MODEL PROPOSAL

The proposed model aims to solve the distribution problem of finished products to meet the demand of the external market through a capacitated minimum cost flow model while respecting all constraints of the problem.

This model has 4 indexes, where i represents the origin, j the destination, p the transported product, and m the mode of transportation. As for the parameters, they are as follows: C_{ijpm}, representing the cost in R$/ton of transporting product p from origin

i to destination j via transportation mode m; b_{ip}, representing the quantity in tons of supply or demand at node i for product p; u_i representing the maximum export capacity in tons of each port (i). As for the decision variable, we have X_{ijpm}, representing the quantity transported in tons from origin i to destination j for product p via transportation mode m.

The mathematical transportation model is as follows:

Objective Function:

$$Min\,Z = \sum_{i=1}^{I}\sum_{j=1}^{J}\sum_{p=1}^{P}\sum_{m=1}^{M}(C_{ijpm} . X_{ijpm}) \quad (1)$$

Subject to:

Factories:

$$\sum_{j=1}^{J}\sum_{m=1}^{M}(X_{ijpm}) = b_{ip} \quad (2)$$

i: (1, ... ,18); p: (1,2,3); j: (19, ... ,24); m: (1,2);

Warehouses:

$$\sum_{j=1}^{J}\sum_{m=1}^{M}(X_{ijpm}) - \sum_{k=1}^{J}\sum_{m=1}^{M}(X_{kipm}) = b_{ip} \quad (3)$$

i: (19,20); p: (1,2,3); j: (21, ... ,24); k: (1, ... ,18); m: (1,2);

Port Terminals:

$$\sum_{j=1}^{J}\sum_{m=1}^{M}(X_{ijpm}) - \sum_{k=1}^{J}\sum_{m=1}^{M}(X_{kipm}) = b_{ip} \quad (4)$$

i:(21, ... ,24); p: (1,2,3); j: (25, ... ,31); k: (1, ... ,20); m: (1,3);

M.E. Clients:

$$-\sum_{k=1}^{J}\sum_{m=1}^{M}(X_{kipm}) = -b_{ip} \quad (5)$$

i: (25, ... ,31); p: (1,2,3); k: (21, ... ,24); m: (3);

$$0 \le X_{ijpm} \le u_i \quad (6)$$

The proposed model consists of several equations that perform specific functions. The objective function (Equation 1) aims to minimize the total distribution cost of the company by calculating the sum of route costs multiplied by the quantity transported, in Brazilian Reais (R$). The set of constraints (Equation 2) ensures that the total quantity of goods sent from factories to warehouses and port terminals does not exceed the production capacity of the factories for the three products. The constraints (Equation 3) balance the quantity produced with demand, allowing only the entry or exit of products from warehouses as needed. The constraints (Equation 4) ensure that the total quantity of each product arriving at the port terminals equals the quantity leaving, allowing only the change of transportation mode from road to maritime. Finally, the constraints (Equation 5) ensure that the demand of each customer for each product is met, equating the quantity received to the quantity purchased. Equation 6 prevents negative quantities of goods transported and limits the total capacity of each port, promoting diversification of the company's transport partners among the 4 mentioned ports.

5.1 RESULT OF MODEL APPLICATION

An experimental application was conducted with a reduced amount of supply, transshipments, and external markets to validate and facilitate the analysis of the model's effectiveness. The LINGO software and Microsoft Excel were used to obtain the results.

The mathematical model presents 264 possible routes, with 18 origins, 2 warehouses, 4 transshipments, 7 destinations, 3 products, and 3 modes of transportation. However, not every origin produces all three products, and each mode is designated for specific routes: mode 1, container truck, is only used from factories to port terminals and, if necessary, from warehouses to ports; mode 2, refrigerated truck, is only used from factories to warehouses; mode 3, container ship, is only used from ports to external market customers on maritime routes.

The result of the objective function obtained by LINGO was: 0.1478983 x 108, representing the optimal solution to the problem, which amounts to a total cost of R$ 14,789,830.71. The LINGO software identified that the solution found by the proposed model is a unique and optimal solution that meets all imposed restrictions and requirements. In other words, the total cost value represents the minimum total cost for the export process with the values and data of this example.

Next, the optimal result of the decision variables of the model is presented only for product 1 since the same procedure will be applied to products 2 and 3. Table 9 shows the result of the routes and quantities to be transported for product 1 from factories to warehouses, in tons.

Table 9. Optimal route for product 1: from factories to warehouses, in tons.

Product 1						
Route	**Departure from Factory**	**Entry to Warehouse**	**Quantity (in tons)**	**Price (route/ton)**	**Total Price of Route**	**Number of Trucks**
X12012	*Node 1 (GO)*	*Node 20 (Itajaí)*	*10,030*	*R$ 16.51*	*R$ 165.595.30*	*359*
X22012	*Node 2 (GO)*	*Node 20 (Itajaí)*	*19,852*	*R$ 17.96*	*R$ 356.541.92*	*709*

Table 10 presents the result of the routes and quantities to be transported of product 1 from the factories to the port terminals, in tons.

Table 10. Optimal route for product 1: from factories to ports, in tons.

Product 1						
Route	**Departure from Factory**	**Entry to Warehouse**	**Quantity (in tons)**	**Price (route/ton)**	**Total Price of Route**	**Number of Trucks**
X22311	*Node 2 (GO)*	*Node 23 (Paranaguá)*	*5,772*	*R$ 34.48*	*R$ 199,018.56*	*241*
X42211	*Node 4 (GO)*	*Node 22 (Navegantes)*	*19,852*	*R$ 34.33*	*R$ 681,519.16*	*823*
X52311	*Node 5 (MG)*	*Node 23 (Paranaguá)*	*4,821*	*R$ 29.69*	*R$ 143,135.49*	*201*
X62211	*Node 6 (MS)*	*Node 22 (Navegantes)*	*7,204*	*R$ 29.84*	*R$ 214,967.36*	*301*
X62311	*Node 6 (MS)*	*Node 23 (Paranaguá)*	*5,165*	*R$ 29.56*	*R$ 152,677.40*	*216*
X72411	*Node 7 (MT)*	*Node 24 (Rio Grande)*	*39,594*	*R$ 40.00*	*R$ 1,583,760.00*	*1,650*
X92111	*Node 9 (PR)*	*Node 21 (Itajaí)*	*4,932*	*R$ 11.73*	*R$ 57,852.36*	*206*
X92311	*Node 9 (PR)*	*Node 23 (Paranaguá)*	*11,902*	*R$ 10.78*	*R$ 128,303.56*	*496*
X102111	*Node 10 (PR)*	*Node 21 (Itajaí)*	*63,694*	*R$ 15.26*	*R$ 971,970.44*	*2,654*
X112111	*Node 11 (PR)*	*Node 21 (Itajaí)*	*13,661*	*R$ 23.05*	*R$ 314,886.05*	*570*
X122111	*Node 12 (RS)*	*Node 21 (Itajaí)*	*27,968*	*R$ 14.37*	*R$ 401,900.16*	*1,166*
X132111	*Node 13 (RS)*	*Node 21 (Itajaí)*	*15,110*	*R$ 13.27*	*R$ 200,509.70*	*630*
X142111	*Node 14 (RS)*	*Node 21 (Itajaí)*	*12,605*	*R$ 14.61*	*R$ 184,159.05*	*526*
X162111	*Node 16 (SC)*	*Node 21 (Itajaí)*	*22,249*	*R$ 12.15*	*R$ 270,325.35*	*927*
X172111	*Node 17 (SC)*	*Node 21 (Itajaí)*	*12,771*	*R$ 15.31*	*R$ 195,524.01*	*533*
X182111	*Node 18 (SC)*	*Node 21 (Itajaí)*	*12,771*	*R$ 12.12*	*R$ 154,784.52*	*533*

Table 11 presents the results of the routes and quantities to be transported for product 1 from port terminals to external market clients, in tons.

Table 11. Optimal route for product 1: from port terminals to external market clients, in tons.

Product 1						
Route	**Departure from Factory**	**Entry to Warehouse**	**Quantity (in tons)**	**Price (route/ton)**	**Total Price of Route**	**Number of Trucks**
X212613	*Node 21 (Itajaí)*	*Node 26 (Americas)*	*10,786*	*R$ 15.68*	*R$ 169,124.48*	*450*
X212913	*Node 21 (Itajaí)*	*Node 29 (South Pacific)*	*9,516*	*R$ 16.57*	*R$ 157,680.12*	*397*
X213113	*Node 21 (Itajaí)*	*Node 31 (Middle Eats)*	*165,459*	*R$ 22.06*	*R$ 3,650,025.54*	*6,895*
X223013	*Node 22 (Navegantes)*	*Node 30 (Japan)*	*27,056*	*R$ 30.94*	*R$ 837,112.64*	*1,128*
X232513	*Node 23 (Paranaguá)*	*Node 25 (Africa)*	*14,002*	*R$ 19.85*	*R$ 277,939.70*	*584*
X232713	*Node 23 (Paranaguá)*	*Node 27 (Europe)*	*13,658*	*R$ 16.72*	*R$ 228,361.76*	*570*
X242813	*Node 24 (Rio Grande)*	*Node 28 (Far East)*	*21,970*	*R$ 26.52*	*R$ 582,644.40*	*916*
X243113	*Node 24 (Rio Grande)*	*Node 31 (Middle East)*	*17,624*	*R$ 24.60*	*R$ 433,550.40*	*735*

The results obtained meet all the constraints of the maximum capacity problem and the quantities required by each node for incoming and outgoing goods. With this solution, no client would fail to receive their goods, the factories withdraw all that has been produced, the port terminals export everything they receive, not exceeding the capacity required for each one, and the warehouses receive part of the demand, balancing the flow.

6 Discussion

The successful implementation of the proposed mathematical model for optimizing the exportation process yields significant results for the company under study. Previously, the company faced challenges in decision-making due to the complexity and volume of data involved. With analyses conducted by independent sectors, determining the optimal solution to minimize total costs was a time-consuming and often subjective task.

However, with the application of the mathematical model and the assistance of specialized software, the company now enjoys a range of practical benefits. Knowing the specific transport cost associated with each route results in more assertive decision-making and a more accurate risk management system, based on the hierarchization of priorities, enabling focus on routes that move most products. For example, Route X72411 has a value of R$ 1,583,760.00, which is approximately four times higher than the average. This means that it should be the top priority for the company when it comes to employee attention, backup transport trucks, and any potential impacts from external factors that could harm this transport route.

The model allows for finding the solution resulting in the lowest total distribution cost, providing an objective and efficient approach to managing the exportation process. Furthermore, the use of the model significantly reduces the time required to analyze scenarios and make decisions, offering a quicker and more effective response to market demands [11][12]. The implementation of the model results in more timely deliveries, and possible price reductions, and ensures that the demanded quantity is delivered as agreed, providing a better experience for the end customer [13][14].

This study contributes to the academic literature by presenting an effective approach to optimizing the exportation process in a food company. The results obtained can serve as a basis for future research and studies on the subject. In addition to the direct benefits for the company under study, the successful implementation of the model demonstrates the potential of mathematical optimization techniques in solving real-world complex problems, offering valuable insights for other organizations facing similar challenges [15][16].

7 Final Considerations

The present study achieved its general objective by proposing a mathematical model for optimizing the logistics network of the export process of the studied food company. Through in-depth theoretical research on operational research concepts and their applicability in input distribution, it was possible to develop a model that found an optimal solution with the lowest total cost.

To evaluate the fulfillment of the proposed objectives, several measures were taken. Firstly, a comprehensive data survey was conducted in various areas of the company, mapping the main parameters and variables of the logistics network for distributing finished products to external market customers. This enabled the formulation of the mathematical model, using the theory of minimum cost flow and operational research knowledge, aiming for its applicability with different datasets each month.

The application of real company data to the proposed model was done using the LINGO software, which analyzed the generated results and confirmed the existence of an optimal and unique solution to the problem. The results were organized into tables presenting the best export routes for products *X*, *Y*, and *Z*, indicating the origin and destination nodes, route prices, and necessary transport modes, among other relevant details.

As a suggestion for future research, expanding the model to consider more variables and company parameters, including other relevant operational constraints, is recommended. Additionally, a post-optimality analysis of the model through sensitivity and parametric analyses may provide additional insights to further enhance the efficiency of the export process.

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