

Comparative Qualitative Evaluation of the Traditional Shewhart Chart and EWMA Applied in a Food Industry

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Abstract. Statistical Process Control (SPC) plays a fundamental role in ensuring quality and production efficiency, allowing the identification and correction of shifts in the process, contributing to customer satisfaction and cost reduction. Thus, the main objective of the study is to carry out a qualitative comparative analysis of the use of traditional Shewhart control charts for mean and amplitude and Exponentially Weighted Moving Average (EWMA), in terms of practical significance, to monitor the packaging process, aiming to assist decision making regarding product conformity and quality, such as reducing product losses and preventing non-conformities. To achieve this objective, the case study research method was used and the assessment method adopted was the comparative analysis of the use of control charts for mean and amplitude with the EWMA control chart. Analysis of the results, focusing on the interpretation of the graphs obtained, revealed that the packaging process did not remain under control during the sample collection period. Thus, the study demonstrates that the application of SPC, through EWMA control charts, is a more sensitive approach to monitoring small changes in the process. However, in terms of practical significance, intervention to correct them may be less advantageous. The methodology adopted for the qualitative comparative assessment provides guidance for decision-making regarding the quality and efficiency of production in the food industry, from the perspective of practical significance.

Keywords: Statistical Process Control, Shewhart Charts, EWMA Control Chart.

1 Introduction

The quality of products and services consumed is an increasingly competitive factor in the market, enabling organizations to have advantages over others. Quality assurance has, therefore, been incessantly sought by companies that aim to sustain themselves in the market with differentiators that can win over customers. In this regard, Statistical Process Control (SPC) presents itself as a tool capable of enabling process monitoring for defined control variables, so that control charts help in visualizing the process behavior. The basic foundation of SPC is based on the concepts of variability [1]-[2]. Adding the different definitions for quality to the concept of variability, there is a common bias that justifies the application of SPC and its practical significance that allows the improvement of the process, since the problems that are identified generate

plans of actions that aim to eliminate or reduce important causes of variability in the process [3]-[5]. In this way, the use of traditional Shewhart control charts (\bar{X} and R) and Exponentially Weighted Moving Average (EWMA) play a fundamental role in the area of quality and process management.

The \bar{X} and R control charts are used to monitor the mean and variation of production processes, being particularly effective in detecting sudden deviations, and therefore, are more suitable for processes that maintain a level of stability [6]. On the other hand, EWMA charts are capable of identifying trends and more gradual variations, adapting sensitively to processes in which changes occur slowly over time, making them ideal for scenarios that require frequent and continuous adjustments to optimization [7]-[8].

Some studies compare the use of the \bar{X} control chart with the EWMA control chart, in terms of finding trends, as [9], also referring to interdependent and independent evaluation of the data as [10], and shifts of data as [11]. Additionally, [12] explore the use of the EWMA control chart through a systemic literature review, highlighting the lack of research that uses control charts with practical applications in real-world data.

Therefore, the purpose of this study is to examine and validate the use of the EWMA control chart in comparison with a mean and amplitude control chart, using real data from a food industry. This aims to fill the gap in the application of EWMA charts in contexts involving concrete information and the practical significance of statistical monitoring for these control charts. The objective is to support production monitoring and control decisions, resulting in benefits such as improving food safety, quality of food products, greater added value to the items produced, and, as a consequence, strengthening economic, social and environmental development.

2 Theoretical Review

2.1 Traditional Shewhart Control Chart

Statistical process control is a sampling inspection process, which must operate throughout the process, with the objective of verifying the presence of special causes, that is, causes that are not natural to the process and that may harm the quality of the product. The SPC provides an overview of the process, identifying its variability and enabling control of this variability over time through continuous data collection, analysis and blocking of possible special causes that are making the system unstable. Quality control began in the 1920s in the United States because of advances in measurement technology and the industrial application of the \bar{X} and R control charts, developed by Dr. Walter A. Shewhart, of the Bell telephone company. Telephone Laboratories. Dr. Walter Shewhart developed a simple but powerful technique for distinguishing between common causes and special causes: process control charts [13].

Control charts have become an important tool for monitoring the quality and the production process in industries. The control charts are designed and used in manufacturing and service industries to detect a shift in the process to minimize the non-conforming product. In the modern era, it has become essential to design a more efficient control chart to watch a tiny shift in the process [14]. According to [15], the SPC is used to analyze information about sample products and to make decisions about processes. Such statistical approaches are crucial for quality assurance. This statistical method provides the main means of sampling, testing, and evaluating a product, and information from these data is utilized to control the manufacturing process and improve it.

The Shewhart control charts are simple in designing and application, so they are widely used in the industries. However, existing Shewhart charts are unable to detect a small shift [14] and it presents low sensitivity to detect subtle changes [16]. In this way, it is necessary to study the process to implement a control chart that is efficient and appropriate for it, to detect variations that affect the quality of products and processes. So, in this way in this way, testing more sensitive graphics for process analysis can improve quality control and meet customer needs.

There are some processes which, owing to their nature, are expected to have

unavoidable shifts in their average value but are still able to satisfy customer-established specifications. What is desired in this case is protection against a situation where the process may shift so far from the target value that it will produce some non-conforming products. If the capability standard deviation is considerably smaller than one-sixth of the tolerance width, then one should consider the use of acceptance charts [17].

According to [17], acceptance chart is used in variable acceptance sampling. When the upper and lower specifications are known, and the standard deviation is unknown, the acceptance region chart allows to see the region of sample means and standard deviations for which the lot will be accept. Duncan defines three types of “acceptance control charts”: i) Projects based on rejectable process level (RPL) and beta risk; ii) Projects based on an acceptable process level (APL) and alpha risk; iii) Projects based on RPL, APL, beta, and alph. There are three different ways to set limits, but there is only one type of chart: a chart that allows you to detect, using X_s for example, when the process is likely to be producing an unacceptable level of nonconforming product. One way is using EWMA, by proposing that the X_s be replaced by EWMA. This approach will allow the accept/reject decision to be made as each data point is accumulated. The EWMA control charts will be discuss in the next section.

2.2 Exponential Weighted Moving Average (EWMA) Control Chart

The Exponential Weighted Moving Average (EWMA) control chart is an alternative to the Shewhart chart, with the capability to detect small deviations in the process mean [18]. Introduced by Roberts in 1959, it is useful for identifying persistent changes in a process. A significant advantage of EWMA is its ability to swiftly detect small shifts, resulting in numerous publications on monitoring process variation using this chart [19]-[20].

According to [21], EWMA is typically used with individual observations, considering that the most recent observation carries a different weight than the previous ones. The statistic representing this concept is denoted as Y_i and can be defined as:

$$Y_i = \lambda X_i + (1 - \lambda)Y_{i-1} \quad (1)$$

$$\text{Containing variance } \sigma_{Y_i}^2 = \sigma^2 \left(\frac{\lambda}{2-\lambda} \right) [1 - (1 - \lambda)^{2i}] \quad (2)$$

In which, $0 < \lambda \leq 1$, and $Y_i = \mu_0$ represents the mean value under control of X , and σ^2 symbolizes the variance of the variable X . In this context, the centerline and k -sigma limits of the EWMA chart are expressed by:

$$\widehat{UCL} = \mu_0 + k\sigma_0 \sqrt{\left(\frac{\lambda}{2-\lambda} \right) [1 - (1 - \lambda)^{2i}]} \quad (3)$$

$$\widehat{CL} = \mu_0 \quad (4)$$

$$\widehat{LCL} = \mu_0 - k\sigma_0 \sqrt{\left(\frac{\lambda}{2-\lambda} \right) [1 - (1 - \lambda)^{2i}]} \quad (5)$$

As the parameter λ approaches 1, the sample values converge towards the values observed on the Shewhart control chart. Typically, values of $k = 3$ and $\lambda = 0.2$ are commonly used [8].

The parameter σ_0 represents the standard deviation of the process when it is in control. According to [21], as i increases, the quantity $[1 - (1 - \lambda)^{2i}]$ tends toward unity, and the chart limits tend to converge. Thus, the control limits can be expressed as:

$$\widehat{UCL} = \mu_0 + k\sigma_0 \sqrt{\left(\frac{\lambda}{2-\lambda} \right)} \quad (6)$$

$$\widehat{LCL} = \mu_0 - k\sigma_0 \sqrt{\left(\frac{\lambda}{2-\lambda} \right)} \quad (7)$$

The authors [12] analyze the EWMA control chart through a systematic literature review, indicating the need for further studies that employ control charts with real data applications and practical significance.

2.3 Process Capability Indices

The assessment of industrial process capability is crucial to ensure production quality and avoid additional costs. Considering the influence of special causes on process capability is a critical point. Adhering to specification limits is essential to avoid penalties and ensure the proper use of products in the industry.

The capability can be measured by the ratio of specification tolerance to the natural process variability represented by 6σ . In practice, σ is estimated as $\hat{\sigma} = \frac{\bar{R}}{d_2}$ [13]. This assessment should only be conducted after the process has stabilized and undergone no interventions during data collection to maintain the natural variability of the process [22].

There are several indices to measure process capability. According to [23], the most commonly used are Cp and Cpk, defined and estimated, respectively, as:

$$Cp = \frac{USL-LSL}{6\sigma} \quad \hat{Cp} = \frac{USL-LSL}{6\hat{\sigma}} \quad (8)$$

$$Cpk = \text{Min} \left\{ \frac{USL-\bar{X}}{3\hat{\sigma}}, \frac{\bar{X}-LSL}{3\hat{\sigma}} \right\} \quad (9)$$

Where, σ represents the natural standard deviation of the process, Upper and Lower Specification Limits are denoted as USL and LSL, \bar{X} and represents the mean. In any of these formulations, the higher the index, the better the process capability; however, conventionally, the minimum acceptable value is 1.0. A Cpk equal to 1.33 is considered good, indicating that the process mean is 4 standard deviations away from the nearest specification limit [24]. Regarding these indices, Cp only makes sense when the process is centered on the nominal value. Cpk allows evaluating whether the process is achieving the nominal value, as it takes into account the mean value [13].

2.4 Process Variability and Practical Significance

As exposed, in all processes, the presence of variability is a reality, as stated by study [25]. This variability may stem from the intrinsic nature of the process or be associated with losses, inefficiencies, and lack of control. The authors [26] emphasize that in a production process, variation is inevitable, even for perfect measuring instruments that do not exist in practice. Variability is evident in samples collected during the production process, where the same measured item can yield different values compared to the nominal. This variability can be expressed by repeatability (agreement in successive measurements under the same conditions) and reproducibility (agreement in measurements in different situations, such as different production shifts).

It is crucial to identify and analyze the causes of variability, especially when not natural, to establish control mechanisms to meet the key product quality characteristics [13]. In manufacturing processes, significant parameter changes are common in early stages, but it is crucial to identify variations in product quality and classify them as Common or Random Causes versus Special or Assignable Causes [26].

Effective variability management involves distinguishing between Common Causes, inherent to the process and treatable with standardized methods, and Special Causes, which are sporadic variations requiring intervention to maintain statistical control. The author [4] underscores the importance of closely monitoring experimental runs to prevent occurrences of special causes. In summary, understanding and managing variability are crucial elements to ensure stability and quality in the manufacturing process.

3 Methodology

In the context previously presented regarding the control charts for mean and range (\bar{X} -R) and EWMA, a case study was chosen as a research method to investigate and analyze the specific operation of a refined sugar packaging company in practical terms. The aim was to understand how the company handles the packaging of 6-gram sugar sachets and ensure that the process is effective and efficient.

Therefore, the case study is a valuable tool as it allows for an in-depth analysis of a real-life case in its natural context. This method aids in exploring complex issues and understanding how various elements interact with each other. By focusing on a specific case, researchers can identify patterns, trends, and peculiarities that may not be as visible in broader research [27].

Furthermore, the case study can be useful for investigating situations that cannot be controlled or manipulated, such as historical events, specific organizations, individuals, or natural processes. Through data collection and observations, a detailed narrative of the case at hand can be constructed. The application of the case study may vary according to the context and the research objective and can be used to explore a wide range of topics, including practical studies in organizations and businesses [28].

Thus, the case study conducted in this research focuses on a company in the food sector specializing in the packaging of refined sugar, with an emphasis on its production line dedicated to packaging 6-gram sachets. The main objective of the study is to qualitatively compare the use of the traditional Shewhart control chart (\bar{X} -R) and EWMA in terms of practical significance for this production line. This is done with the purpose of preventing product losses and non-conformities related to products with weights below the indicated value.

The steps for monitoring the production line adopted in this research are described in detail as follows:

Step 1: Selection of the Study Context. The choice of the refined sugar packaging company and its production line of 6 grams sachets was made due to the relevance of the sector and the importance of ensuring the quality and compliance of products in this process.

Step 2: Sampling Frequency. To conduct the research, a sampling frequency of 15 minutes was established, where a set of 5 samples with 5 items each was collected at each 15-minute interval.

Step 3: Sample Size. Each sample consisted of 5 sachets, composing 25 values, providing a comprehensive view of the packaging operations in that period.

Step 4: Evaluation Method. The evaluation method adopted was a graphical comparison of the traditional Shewhart control chart for Mean and Range (\bar{X} -R) with the EWMA.

Step 5: Results Analysis. After collecting the samples and applying the control charts, the results were recorded and analyzed. Emphasis was given to the interpretation of the resulting charts to assess the effectiveness of Statistical Process Control (SPC) in the chosen production process.

Step 6: Study Conclusions. Based on the analysis of the charts and the obtained results, the company can conclude whether the packaging process remained under control during the period in which the samples were collected. This practical significance observation in the control charts is crucial to ensuring the quality and efficiency of the production line, as well as minimizing losses and non-conformities.

The research method employed in this case study provided a systematic approach to effectively monitor, in practical terms, the packaging process, ensuring compliance and product quality. The careful analysis of sample data allowed the company to make informed decisions and implement necessary improvements in the production process.

4 Results and Discussion

The analyzed product is individual sachets of crystal sugar, each weighing 6 grams. The focus of this analysis is the weight (g) of each sachet, a characteristic controlled by consumer protection agencies and ensured for quality by the company. For this study, data on the weight of the sachets were recorded during the filling process for a period of 4 hours daily over 5 days.

The manufacturing process for this product involves several steps. Firstly, sugar is received from the crystallization process. Subsequently, the product reservoir, with a capacity of 1,000 kg, is loaded. The packaging machine is then prepared with packaging film. The filling process is meticulously adjusted, with any out-of-spec

products being discarded.

Upon confirming the film adjustment and product weight, the packaging process ensues. The next step involves the manual positioning of a cardboard box designed to accommodate batches of sachets. Each box is intended to contain 300 sachets, each weighing 6g, resulting in a total product weight of 1,800g per box. This comprehensive process ensures the efficient and accurate manufacturing of the product. The company operates with volumetric dosing machines that measure the quantity of sugar in each sachet. This machine produces 4 sachets with each revolution, and the pre-established cycle is 30 revolutions per minute.

In general, volumetric dosing does not undergo significant changes except when there is a variation in product temperature and humidity. For instance, on a hot and dry day, the dosing machine settings are different from a cold and rainy day. Small variations throughout the day require minor adjustments to the dosing machines. Therefore, to determine reference values, UCL (Upper Control Limit), CL (Center Line), and LCL (Lower Control Limit), considering the 5 subgroups, equations for the control chart for range (1, 2, and 3) were used. The results are shown in the data presented in Table 1, as well as in the control chart shown in Fig. 1.

Table 1. Record of Samples with Range Values UCL, CL, and LCL

Sample	Observations (g / sachet)					R	UCL	CL	LCL
1	6.030	6.017	6.022	6.013	6.014	0.017	0.038	0.018	0.000
2	6.019	6.020	6.020	6.018	6.001	0.019	0.038	0.018	0.000
3	6.014	6.030	6.025	6.023	6.027	0.016	0.038	0.018	0.000
4	6.002	6.018	6.017	6.002	6.030	0.028	0.038	0.018	0.000
5	6.012	6.024	6.023	6.022	6.025	0.013	0.038	0.018	0.000
6	6.014	6.012	6.008	6.002	6.009	0.012	0.038	0.018	0.000
7	6.025	6.030	6.027	6.020	6.015	0.015	0.038	0.018	0.000
8	6.012	6.025	6.019	6.017	6.018	0.013	0.038	0.018	0.000
9	6.008	6.015	6.016	6.020	6.012	0.012	0.038	0.018	0.000
10	6.002	6.023	6.017	6.020	6.024	0.022	0.038	0.018	0.000
11	6.015	6.024	6.020	6.019	6.030	0.015	0.038	0.018	0.000
12	6.030	6.025	6.030	6.027	6.023	0.007	0.038	0.018	0.000
13	6.006	6.011	6.012	6.013	6.008	0.007	0.038	0.018	0.000
14	6.000	6.022	6.002	6.018	6.015	0.022	0.038	0.018	0.000
15	6.002	6.024	6.012	6.020	6.027	0.025	0.038	0.018	0.000
16	6.030	6.018	6.013	6.008	6.012	0.022	0.038	0.018	0.000
17	6.012	6.023	6.012	6.007	6.013	0.016	0.038	0.018	0.000
18	6.023	6.017	6.030	6.001	6.002	0.029	0.038	0.018	0.000
19	6.013	6.018	6.025	6.020	6.027	0.014	0.038	0.018	0.000
20	6.020	6.022	6.014	6.001	6.027	0.026	0.038	0.018	0.000
21	6.027	6.002	6.023	6.030	6.002	0.028	0.038	0.018	0.000
22	6.017	6.025	6.023	6.019	6.018	0.008	0.038	0.018	0.000
23	6.002	6.030	6.014	6.022	6.020	0.028	0.038	0.018	0.000
24	6.023	6.018	6.019	6.017	6.030	0.013	0.038	0.018	0.000
25	6.001	6.002	6.013	6.020	6.022	0.021	0.038	0.018	0.000

R = range; UCL = Upper Control Limit; CL = Center Line; LCL = Lower Control Limit.

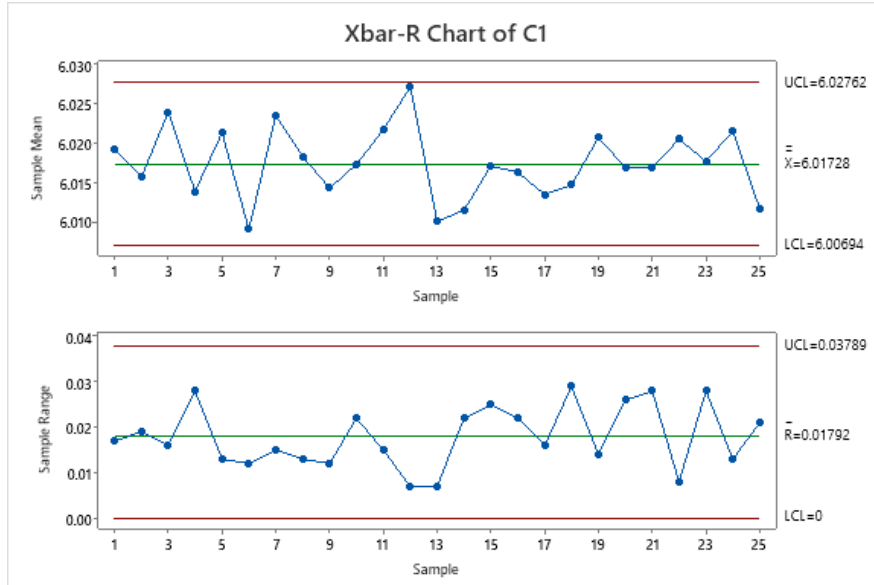


Fig.1. Mean and Range Control Chart.

To establish reference values, UCL, CL, and LCL, considering the 5 subgroups, equations for the control chart for the mean (4, 5, and 6) were employed. The results are shown in the data presented in Table 2 and in the control chart represented by Fig. 1.

Table 2. Record of Samples with Range Values UCL, CL, and LCL

Samples	Observations (g / sachet)	\bar{X}	UCL	CL	LCL
1	6.030 6.017 6.022 6.013 6.014 6.019 6.029 6.018 6.008	6.019	6.029	6.018	6.008
2	6.019 6.020 6.020 6.018 6.001 6.016 6.029 6.018 6.008	6.016	6.029	6.018	6.008
3	6.014 6.030 6.025 6.023 6.027 6.024 6.029 6.018 6.008	6.024	6.029	6.018	6.008
4	6.002 6.018 6.017 6.002 6.030 6.014 6.029 6.018 6.008	6.014	6.029	6.018	6.008
5	6.012 6.024 6.023 6.022 6.025 6.021 6.029 6.018 6.008	6.021	6.029	6.018	6.008
6	6.014 6.012 6.008 6.002 6.009 6.009 6.029 6.018 6.008	6.009	6.029	6.018	6.008
7	6.025 6.030 6.027 6.020 6.015 6.023 6.029 6.018 6.008	6.023	6.029	6.018	6.008
8	6.012 6.025 6.019 6.017 6.018 6.018 6.029 6.018 6.008	6.018	6.029	6.018	6.008
9	6.008 6.015 6.016 6.020 6.012 6.014 6.029 6.018 6.008	6.014	6.029	6.018	6.008
10	6.002 6.023 6.017 6.020 6.024 6.017 6.029 6.018 6.008	6.017	6.029	6.018	6.008
11	6.015 6.024 6.020 6.019 6.030 6.022 6.029 6.018 6.008	6.022	6.029	6.018	6.008
12	6.030 6.025 6.030 6.027 6.023 6.027 6.029 6.018 6.008	6.027	6.029	6.018	6.008
13	6.006 6.011 6.012 6.013 6.008 6.010 6.029 6.018 6.008	6.010	6.029	6.018	6.008
14	6.000 6.022 6.002 6.018 6.015 6.011 6.029 6.018 6.008	6.011	6.029	6.018	6.008
15	6.002 6.024 6.012 6.020 6.027 6.017 6.029 6.018 6.008	6.017	6.029	6.018	6.008
16	6.030 6.018 6.013 6.008 6.012 6.016 6.029 6.018 6.008	6.016	6.029	6.018	6.008
17	6.012 6.023 6.012 6.007 6.013 6.013 6.029 6.018 6.008	6.013	6.029	6.018	6.008
18	6.023 6.017 6.030 6.001 6.002 6.015 6.029 6.018 6.008	6.015	6.029	6.018	6.008
19	6.013 6.018 6.025 6.020 6.027 6.021 6.029 6.018 6.008	6.021	6.029	6.018	6.008
20	6.020 6.022 6.014 6.001 6.027 6.017 6.029 6.018 6.008	6.017	6.029	6.018	6.008
21	6.027 6.002 6.023 6.030 6.002 6.017 6.029 6.018 6.008	6.017	6.029	6.018	6.008
22	6.017 6.025 6.023 6.019 6.018 6.020 6.029 6.018 6.008	6.020	6.029	6.018	6.008
23	6.002 6.030 6.014 6.022 6.020 6.018 6.029 6.018 6.008	6.020	6.029	6.018	6.008
24	6.023 6.018 6.019 6.017 6.030 6.021 6.029 6.018 6.008	6.021	6.029	6.018	6.008
25	6.001 6.002 6.013 6.020 6.022 6.012 6.029 6.018 6.008	6.012	6.029	6.018	6.008

\bar{X} = mean; UCL = Upper Control Limit; CL = Center Line; LCL = Lower Control Limit.

According to [13], even if all points fall within the control limits, meaning between the upper and lower control limits, observing a systematic situation where the points exhibit a special configuration that excludes randomness from the data suggests the process may be out of control. This is because controlled processes are characterized by randomness.

Other factors are also used for this analysis, such as simultaneous points forming

trends either upwards or downwards (i.e., a sequence of random observations), and the sample mean displayed on the chart showing cyclical behavior, even if all points fall within the limits [13].

Based on the above statements, we conclude that the mean and range charts for the observed samples reveal that the process is out of control. They exhibit a slight presence of cyclical behavior from sample 14 onwards, interfering with the randomness of the data's behavior.

The EWMA control chart was created to compare and distinguish itself from another set of mean and range control charts. To determine the values of interest, namely UCL, CL, and LCL, only the individual observations of unit 1 were used, based on equations (3, 4, 5, 6, and 7) as presented in Tabel 3. The moving averages representing the samples were then plotted, resulting in the chart illustrated in Fig. 2, where parameters $\lambda = 0.20$ and $k = 3$ were used.

Table 3. Record of Samples with Range Values UCL, CL, and LCL

	Y_i	UCL	CL	LCL
1	6.0177	6.0151	6.0173	6.0194
2	6.0173	6.0145	6.0173	6.0200
3	6.0186	6.0142	6.0173	6.0203
4	6.0176	6.0140	6.0173	6.0205
5	6.0183	6.0139	6.0173	6.0207
6	6.0165	6.0138	6.0173	6.0207
7	6.0178	6.0138	6.0173	6.0208
8	6.0179	6.0138	6.0173	6.0208
9	6.0172	6.0137	6.0173	6.0208
10	6.0172	6.0137	6.0173	6.0208
11	6.0181	6.0137	6.0173	6.0208
12	6.0199	6.0137	6.0173	6.0208
13	6.0179	6.0137	6.0173	6.0208
14	6.0166	6.0137	6.0173	6.0208
15	6.0167	6.0137	6.0173	6.0208
16	6.0166	6.0137	6.0173	6.0208
17	6.0159	6.0137	6.0173	6.0208
18	6.0157	6.0137	6.0173	6.0208
19	6.0167	6.0137	6.0173	6.0208
20	6.0167	6.0137	6.0173	6.0208
21	6.0167	6.0137	6.0173	6.0208
22	6.0174	6.0137	6.0173	6.0209
23	6.0175	6.0137	6.0173	6.0209
24	6.0183	6.0137	6.0173	6.0209
25	6.0169	6.0137	6.0173	6.0209

Y_i = represents the mean value under control of X ,
UCL = Upper Control Limit, CL = Center Line;
LCL = Lower Control Limit

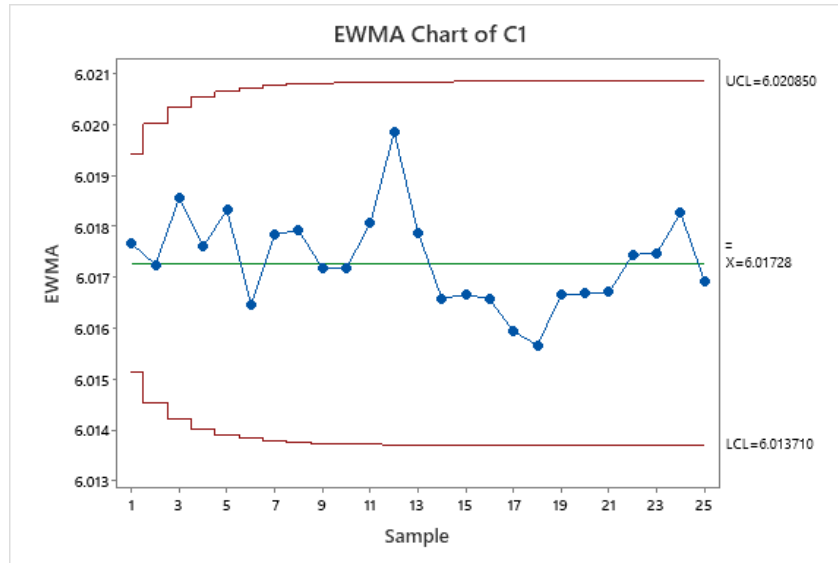


Fig.2. EWMA Control Chart.

In the analysis of the control charts (in Fig.1 and Fig. 2), it is noted that although no points exceed the control limits in the presented figures, the EWMA chart clearly shows a trend starting from the twelfth sample, suggesting that the process may be out of control. This trend is not as clearly observed in the control charts for mean and range (\bar{X} -R), where the twelfth point only approaches the upper control limit without exceeding it, leaving uncertainties regarding the state of control of the process. Only after the fifteenth sample does the control charts for mean and range (\bar{X} -R) begin to exhibit cyclic behavior, which could indicate a process potentially out of control.

Therefore, the EWMA chart proves to be more effective in rapidly detecting changes, providing clearer evidence about the behavior of the process. This suggests that the variations in the process, which may be non-natural and specific, are more noticeable when analyzed through the EWMA chart, due to its sensitivity to small changes in the process, as analyzed in this case study.

Next, process capability indices were calculated, considering that the specification limits for the process are 5.95 and 6.05, as used by the analyzed industry. Therefore, the process has a Cp process capability index of 2.22 and a Cpk of 1.45, as shown in Table 4, indicating that the process is capable and has the ability to meet specifications.

Table 4. Values of Cp and Cpk Process Capability Indices

Cp	Cpku	Cpkl	Cpk
2.22	1.45	2.98	1.45

Cp = process capability index for centered process; Cpku = upper process capability index for non-centered process; Cpkl = lower process capability index for non-centered process; Cpk = process capability index for non-centered process.

However, it is noted that despite the process having high capability, it shows a trend in the EWMA control chart. Therefore, the industry managers were informed about the occurrence, and upon evaluating the implications of the interference that should be performed in the process due to this small shift from the mean—possibly caused by a variation in the measurement system or raw materials—they decided that it would be possible for the process to coexist with it. Interfering would be more costly and would disturb the process further, making it impractical and not meaningful.

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